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T1 ________________	54656	F1 ________________
T2 ________________		F2 ________________
T3 ________________	Problem Chosen	F3 ________________
T4 ________________	A	F4 ________________

2016

Mathematical Contest in Modeling (MCM/ICM) Summary Sheet

A real-life problem is being considered in our article. The problem is about what strategies can be adopted if one wants to take a comfortable bath. The bathtub is modeled as a simple containment vessel which do not have a secondary heating system or circulating jets. However, the temperature of the water can be retained by adding a constant trickle of hot water. Practically, the water in the bathtub will overflow and keep the level if it reaches the overflowing water level. The definition of COMFORTABLE, or the objective of the strategy, is measured by the uniformity of the tub water with regard to time, or, in other words, how can the temperature be retained through time. Another objective for our strategy is to save water as soon as possible. There are bunches of parameters, factors or independent variables that are considered in our model, such as the temperature of hot water being added, the influence of a thin layer of bubble, the volume and shape of the bathtub, the volume, shape and motions of human.

To sum up, our model is aiming at (a) keeping the variation of temperature of the water in the bathtub as smooth as possible and (b) saving as much water as possible. The independent factors being considered are (a) the temperature of hot water being added, (b) the influence of a thin layer of bubble, (c) the volume and shape of the bathtub, and (d) the volume, shape and motions of human.

Two models are build in our article. One is aimed to be as simple as possible, trying to elucidate the relationship between the basic independent variables and our objectives with as little complexity as possible. The other one is meant to be closer to real life, with physical laws closer to nature and simulation with real-life parameters. Dispite the differences of the two models in terms of both physical assumptions and approaches to solve, the conclusion drawn from the two are quite similar in general orientation - obviously, they are different quantitatively. The similar conclusions from these two models gives a powerful mutual validation which lays a solid foundation of forthcoming strategies. Additionally, a non-technical explanation for the users of the bathtub is presented in the end of this article.

Contents

1 Introduction

Prometheus stole the fire and passed it to human. Since then, human gradually learnt the property of fire and the laws of heat. It is no exaggeration to say that human civilization began with the control of fire and the understanding of heat. Today, with a better understanding of the nature of heat trasfer and more advanced mathematical tools, it is still quite interesting to study a very common phenomenon in our real life.

The gradual cooling of water is hard to avoid when bathing in a bathtub without a secondary heating system and circulating jets. However, continuously adding hot water can reheat the water in bathtub and thus keep the temperature in the bathtub more or less. This, on the other hand, will be a kind of wasting water. Our aim is to build a model, give a reasonable strategy upon it, and tell the users of the bathtub how to make a compromise between bathing temperature and water saving.

2 Analysis & symbols

The temperature of the water in the bathtub cools down because it keeps contacting with the surroundings. Considering that the bathtub is a solid, the water is incompressible fluid, and the air is compressible gas, we should analyze them according to different situations. Heat Transfer and Fluid Dynamics are related in solving the problem.

Symbols we will use are listed below.

Symbol	Definition	default value
l_t	The length of the bathtub	2m
h_t	The height of the bathtub	0.6m
l_w	The length of water in the bathtub	1.8m
h_w	The height of water in the bathtub	0.5m
l_a	The length of air	2.5m
h_a	The height of air	2.2m
h_{port}	water inlet and outlet dimension	0.1m
u_i	The speed of hot water	5×10^{-4}m/s
r_{human}	Human radios	0.2m
T_{room}	Room temperature	293K
T_{w0}	The initial temperature of water in the bathtub	313K
T_i	The temperature of hot water added in	318K
h_{t-a}	The heat exchange coefficient of tub-air interface	50W/(m^{2}K)
h_{w-a}	The heat exchange coefficient of water-air interface	1000W/(m^2 *K)
h_{w-t}	The heat exchange coefficient of water-tub interface	50W/(m^2 *K)
T_{human}	Human body temperature	310K
ρ_a	Density of air	1.225kg/m^3
ρ_t	Density of the bathtub	800kg/m^3
ρ_w	Density of water	1000kg/m^3
C_{p_air}	Heat capacity of air	1010J/(kg*K)
C_{p_tub}	Heat capacity of the bathtub	837J/(kg*K)
C_{p_water}	Heat capacity of water	4179J/(kg*K)
k_a	Thermal conductivity of air	20W/(m*K)

k_t	Thermal conductivity of tub	1.22W/(m*K)
k_w	Thermal conductivity of water	0.6W/(m*K)
d_{foam}	Foam thickness	0.05m
k_{foam}	Thermal conductivity of foam	0.031W/(m*K)

3 A simplest model for the problem

3.1 Design inspiration

To solve the problem in the quickest way, we are willing to make a simplest model, in which Newton's law of cooling is the only formula being used. The temperature field in the water is regarded as an even steady-state one, so are the air and the bath, too. Significantly, the temperature of the water is changing as time goes by while the temperature of the air and the bath keeps stable at room temperature.

To build a simplest model, we dip into the problem by beginning at the original case where no man or new water is added in, then we take more aspects into consideration step by step. And that is how we start!

3.2 How this model is built

A diagram of the bathtub with water is presented as in figure 3.2-1, which helps to develop a clear concept of the problem involved.

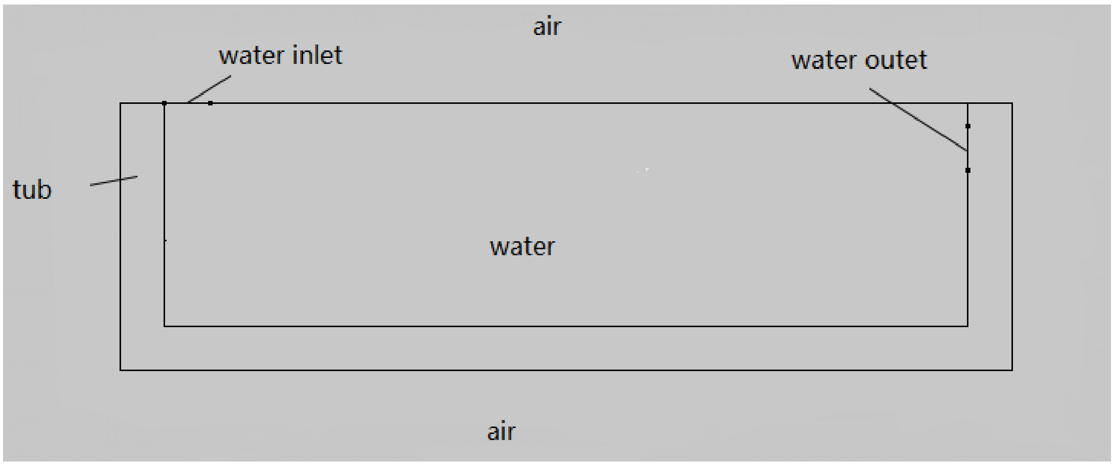

Figure 3.2-1 A diagram of the bathtub

3.2.1 Case one: no man, no new water

In order to construct our ultimate model step by step and make the process of construction explicit, we firstly assume that the simplest model consists of the initial water in the bathtub without new hot water or the bather.

Define $T_w(x,y,z,t)$ as the water temperature, $T_t(x,y,z,t)$ as the bathtub temperature and $T_a(x,y,z,t)$ as the air temperature. We regard $T_t(t)$ together with $T_a(t)$ as the constant home temperature, and the temperature field in the water as an even steady-state temperature field.

According to Newton's law of cooling, the temperature of the water satisfies the following equation:

$$\rho C_p \frac{\partial T_w}{\partial t} V + h_{w-a} S_{w-a}(T_w - T_a) + h_{w-t} S_{w-t}(T_w - T_t) = 0 \quad (1)$$

In this equation, ρ is the density of water, C_p is the heat capacity of water, h_{w-a} and h_{w-t} represent the heat exchange coefficient of water-air interface and water-tub interface respectively. S_{w-a} is the surface area of water-air interface while S_{w-t} is the surface area of water-tub interface, and V represents the volume of water in the tub.

As T_a=$T_t = T_{room}$, where T_{room} is room temperature, we can get:

$$T_w = T_{room} + (T_{w0} - T_{room})e^{-\frac{h_{w-a}S_{w-a}+h_{w-t}S_{w-t}}{\rho C_p V}t} \quad (2)$$

From the last written equation above, we see the temperature of water decreases by e index. Figure 3.2-2 describes how the temperature of water in the tube changes with time.

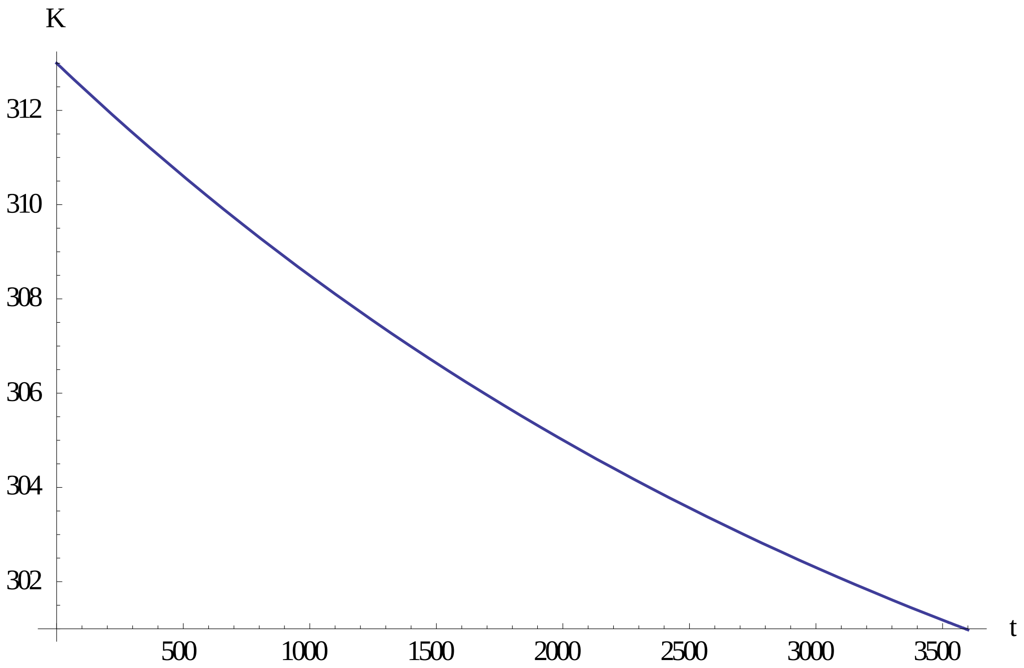

Figure 3.2-2 The changing temperature of water in case one The initial temperature of the water is 313K, the length, height and width of the tub are 2m, 0.6m and 1m. We set h_{w-t} to 50W/(m^2*K) and h_{w-a} to 1000W/(m^2*K).

From figure 3.2-2, it is obviously known that the temperature drops down at a high speed. After an hour, the temperature decreases by more than ten. The bather will feel cold if he does not take any strategies.

3.2.2 Case two: new water in, excess water out

Now the bather adds a constant trickle of hot water from the faucet to reheat the bathing water, then the equation is changed into the following form:

$$\rho C_p \frac{\partial T_w}{\partial t} V + h_{w-a} S_{w-a}(T_w - T_a) + h_{w-t} S_{w-t}(T_w - T_t) = \rho C_p v(T_i - T_w) \quad (3)$$

In this equation, T_i is the temperature of new water added in, while v is the velocity of adding hot water. The formula is based on the energy conservation. In this case, we get:

$$T_w = T_{room} + A\left[\rho C_p v(T_i - T_{room}) - Be^{-\frac{h_{w-a}S_{w-a}+h_{w-t}S_{w-t}+\rho C_p v}{\rho C_p V}t}\right] \tag{4}$$

$$A = \frac{1}{h_{w-a}S_{w-a}+h_{w-t}S_{w-t}+\rho C_p v} \tag{5}$$

$$\mathrm{B} = \rho C_p v(T_i - T_{room}) - (T_{w0} - T_{room})(h_{w-a}S_{w-a} + h_{w-t}S_{w-t} + \rho C_p v) \tag{6}$$

If B=0, that is to say, $\rho C_p v(T_i - T_{w0}) = (T_{w0} - T_{room})(h_{w-a}S_{w-a} + h_{w-t}S_{w-t})$, then $T_w = T_{w0}$, which is constant. But if B>0, the temperature will decrease slowly, otherwise, it will increase gradually until reaching a new constant level.

Let T_i be 318K, to keep the initial temperature, $v = 0.44dm^3/s$, as is shown in figure 3.2-3.

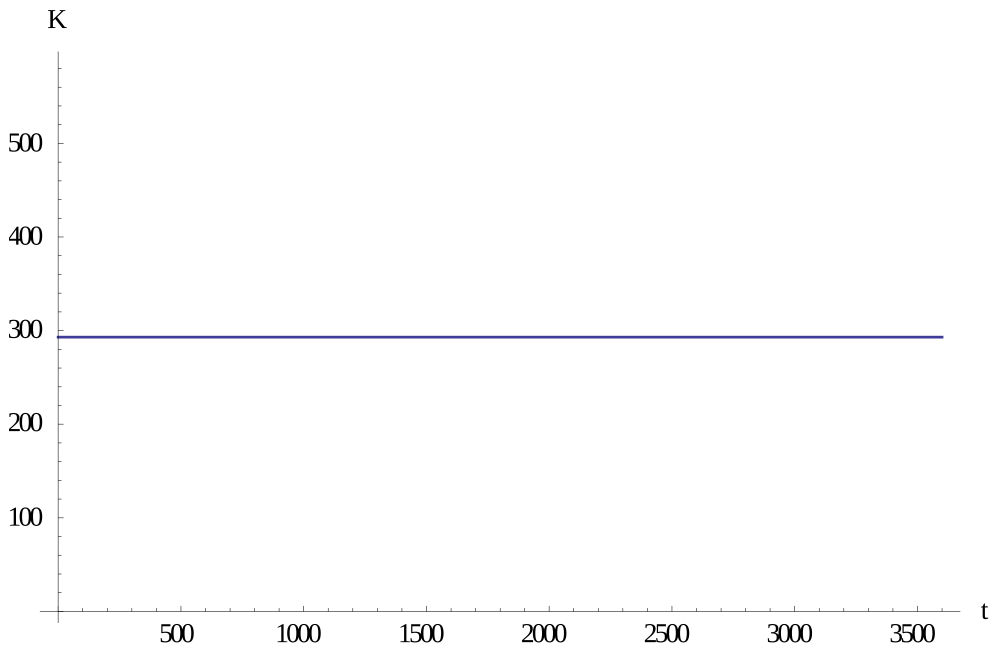

Figure 3.2-3 A strategy of making the water keep its initial temperature. T_i=318K, $v = 0.44dm^3/s$, and the other parameters keeps the same as case one.

3.2.3 Case three: the influence made by the bather

The only influence made by the bather in this simplest model is his volume. When we calculate the water volume, we should minuses the volume of the bather with the volume of the bathtub. The volume of the bather is assumed to be 0.06m^3.

3.3 Conclusions and discussions

This model draws a draft of water cooling and reheating system, which can solve our problem to a considerable extent.

3.3.1 How this model solve the problem

- **The heat exchange coefficient of water-air interface and water-tub interface**

  The heat exchange coefficient of water-air interface is relating to air flow at the interface, so bathing in a large windy area will be at the cost of the fastest cooling, which no one would like to do. As for the heat exchange coefficient of water-tub interface, it depends on the material of tub itself, the only thing we could do may be change a better tub, in order to make it easier to keep the water temperature along with saving water.

- **Influences made by the shape and volume of the bathtub**

  The smaller the tub is, the easier to keep the water temperature. On the other hand, the shape of the bathtub decides the area of interface among water, air and tub.

- **Influences made by the bather**

  The shape and motions of the bather are not within the scope of our consideration in this model. Motions made by the bather just help the water reaches its average temperature without an uneven distribution. The only influence brought by the bather is that he occupies a certain volume, which we should take notice when calculate the water volume in the bathtub.

- **The influence of bubble bath additive**

  Bubble bath additive is used for a filled bathtub with a layer of surfactant foam on the surface of the water. It is so porous that it decreases the heat exchange coefficient of water-air interface remarkably. Consequently, keeping the water temperature becomes much easier than ever before.

3.3.2 A balance between the water temperature and water consuming

The higher of water temperature we want to get, the more hot water we need to consume, as we can see from equation (4)-(6). Figure 3.2-3 serves a probe solution of keeping the initial water temperature. For more details, our next model which is more complex and accurate would discuss it in different aspects.

3.4 Strength and weakness

This model is our first and simplest model, it may not so comprehensive as our next model, but it describes an overall trend of the bathing problem in the shortest time, which helps us develop a rough understanding on it.

However, we would still like to build a second model, a more elaborate model, for a better understanding on temperature field. And the following descriptions are all about how we build it.

4 An elaborate model

In the first place, we design a simplified model of the bathtub, the simulative size of which is the result of plenty of surveys and interviews. Given that it will be shown many times in the following discussion, we provide the vertical section of the bathtub.

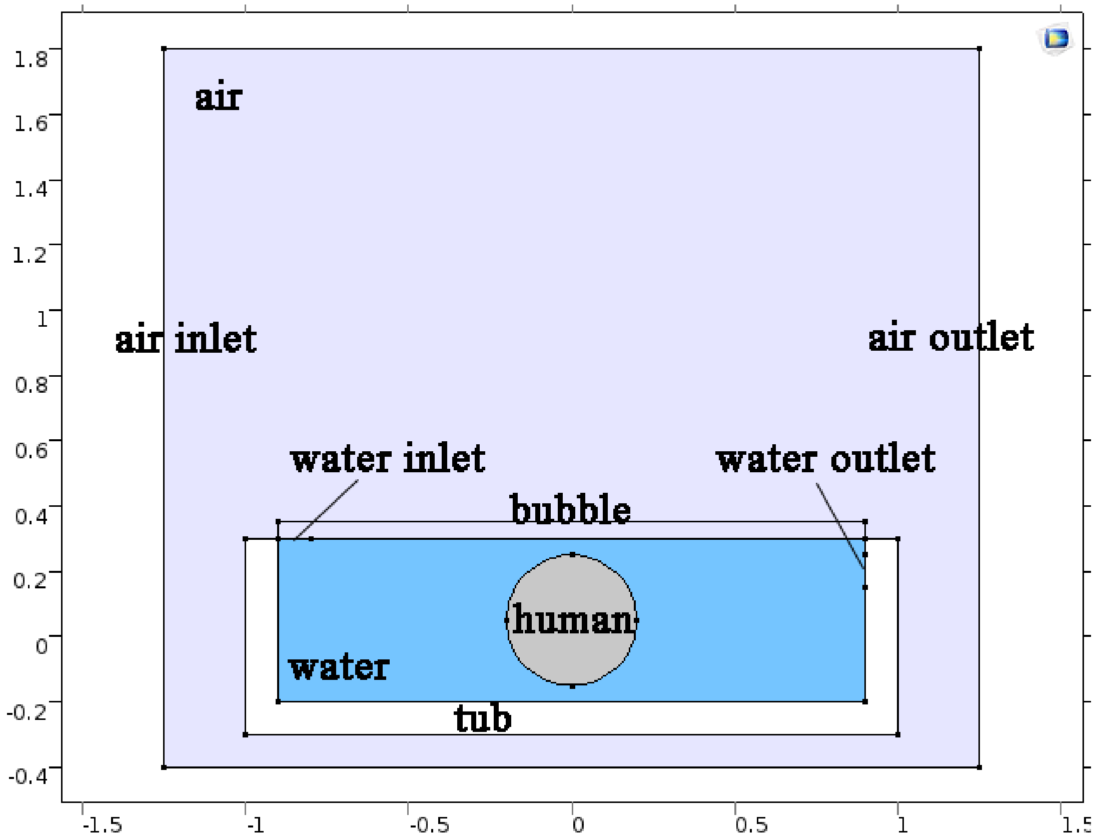

Figure 4-1 A diagram of the model

4.1 The simplest model without change of water, flow or bather

4.1.1 Assumptions and relative theories

In order to construct our ultimate model step by step and make the process of construction explicit, we firstly assume that the simplest model consists of the initial hot water, the bathtub and the air. The water movement, the air movement, the constant trickle of hot water from the faucet and consequently the water wasted, the person and the situation that he or she uses a bubble bath additive are all unaccounted.

In addition, we simply regard the system as a solid-heat conduction system.

The heat conduction equation, the mathematical representation of which is

$$\rho C_\rho \frac{\partial T}{\partial t} = \nabla \cdot (k\nabla T)$$

, is tacitly approved, as it is an empirical formulas suitable for the heat conduction of solid, and is accepted by the majority and applied to all walks of life.

4.1.2 Model theory

We set an initial water temperature at first, 313K for instance, and think the air temperature as room temperature 293K. As for the temperature of the bathtub, we simply think it as the arithmetic mean value of them.

So, the boundary condition is comprised of two conditions. The Dirichlet boundary condition $T=T_{room}$ should be used as outer boundary condition, corresponding with the definition above. Another boundary condition

$$\begin{cases} \vec{n_1} \cdot k_1 \nabla T_1 = -h(T_2 - T_1) \\ \vec{n_2} \cdot k_2 \nabla T_2 = -h(T_1 - T_2) \end{cases}$$

is appropriate for contact surfaces between the air and the tub, the tub and the water and the air and the water as they are all thermal-contact surfaces. h represent heat exchange coefficient and k represent thermal conductivity. For example, the heat exchange coefficient between the air and the tub is precisely 50W/(m^2·K) and the thermal conductivity of the water is 0.6W/(m·K).

The temperature field of the system——a variation that obviously varies over time——is the major object we intend to describe. Three hours later, the temperature field of the system is characterized by the following figure, in which the trait of the temperature field is conspicuous.

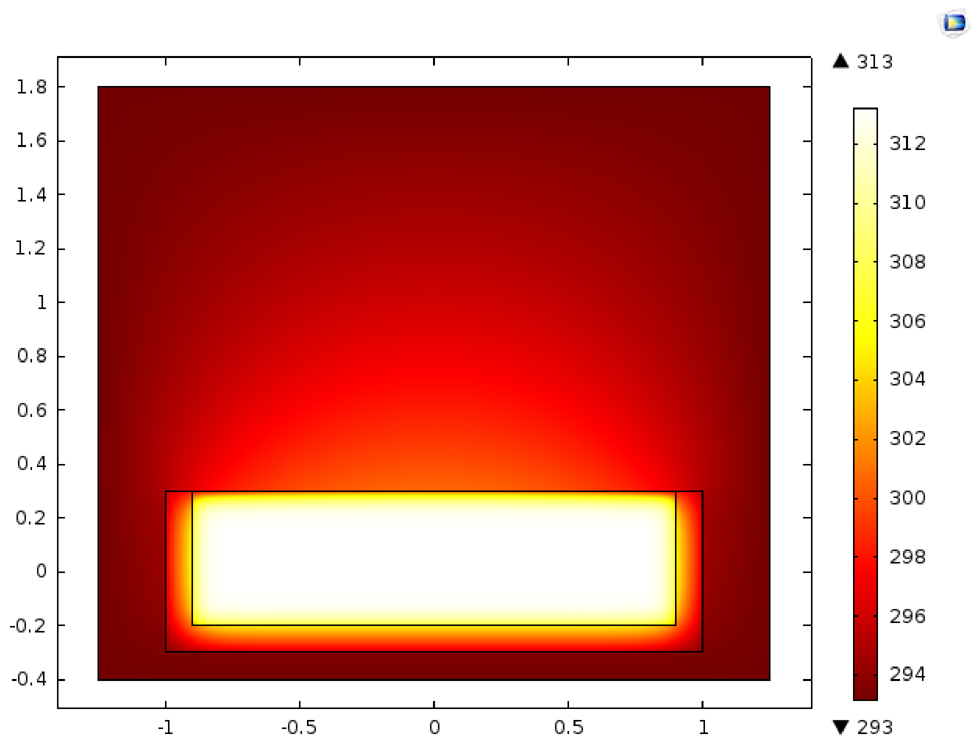

Figure 4.1.2-1 The temperature field 3 hours later

The heat conduction and the natural cooling of water predict that over time, the water temperature will decline and reach room temperature ultimately, along with the system reaching a steady state. It is testified accurately by simulation.

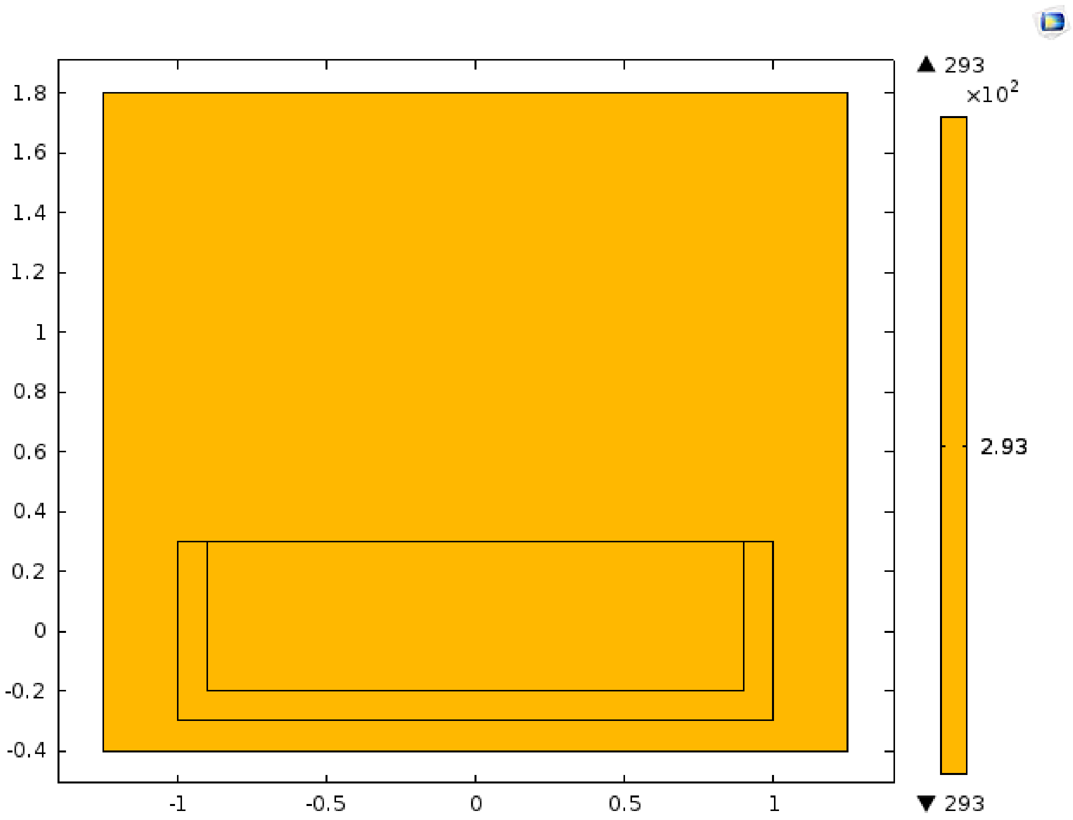

Figure 4.1.2-2 The final temperature field

Since our first model intends to reveal the rule of natural cooling, how the water changes over time should be shown exactly. According to our simulation , the next figure shows a veracious relationship between time and the average of the water temperature.

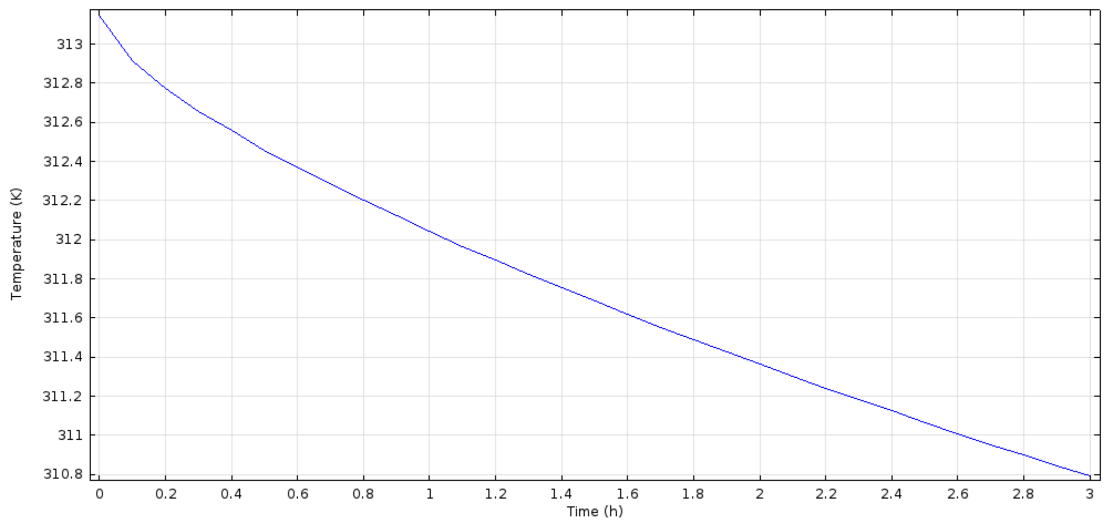

Figure 4.1.2-3 The relationship between time and the average of the water temperature.

However, it seems clash with everyday experience by telling us the water temperature nearly has a linear decline, far from our common sense. Therefore, instead of regarding the system as a solid-heat conduction system, we should take the properties of fluids into account. That is why the second model seems ready to come out at our call.

4.1.3 Remarks

Indeed, this model corresponds with our prediction approximately. While it is oversimplified and is regarded as a solid-heat conduction system, the water temperature declines slower than that we think, as the water flow makes great contribution to heat transmission.

4.2 The second model

4.2.1 Assumptions and relative theories

Compared with the simplest model, this one goes further by considering the flow of air and the flow of water. The constant trickle of hot water from the faucet and consequently the water wasted, the person and the situation that he or she uses a bubble bath additive are all unaccounted.

The useful formulas include the heat equation

$$\rho c_p \frac{\partial T}{\partial t} + \rho c_p \vec{u} \cdot \nabla T = \nabla \cdot (k \nabla T)$$

and fluid equation

$$\begin{cases} \nabla \cdot [-pI + \mu(\nabla \vec{u} + (\nabla \vec{u})^T)] + F = 0 \\ \nabla \cdot \vec{u} = 0 \end{cases}$$

. μ means dynamic viscosity. Pressure p and velocity u are dependant variables.

4.2.2 Model theory

For the bathtub,

$$\rho c_p \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T)$$

. As we consider water as incompressible fluid,

$$\begin{cases} \rho c_p \frac{\partial T}{\partial t} + \rho c_p \vec{u} \cdot \nabla T = \nabla \cdot (k \nabla T) \\ \vec{u} = (circ_u, circ_v) \end{cases}$$

. And as we consider air as compressible fluid,

$$\begin{cases} \rho c_p \frac{\partial T}{\partial t} + \rho c_p \vec{u} \cdot \nabla T = \nabla \cdot (k \nabla T) \\ \nabla \cdot [-pI + \mu(\nabla \vec{u} + (\nabla \vec{u})^T)] + F = 0 \\ \nabla \cdot \vec{u} = 0 \end{cases}$$

. As we can see, in order to make the water homogeneous, different from the first model, an eddy current field that stirs more violently in the center than on the periphery is introduced. Furthermore, it is designed as a ellipse in order to match the shape of the tub. The flux is zero when comes to a sealing surface but is not zero when the surface is ringent.

It is rational to ignore the initial conditions if we only care the steady state to which the system will reach as long as the time is long enough. We need to focus on three boundary conditions. For the air flowing from the left margin,

$$\nabla \cdot [-pI + \mu(\nabla \vec{u} + (\nabla \vec{u})^T)] + f_{flow} = 0$$

. Determined by the viscidity of the fluid, the initial velocity the air need, the pressure and the velocity of the fluid, f_{flow} represents the force required per unit volume when the air is forced to the fluid. For the air flowing out of the system through the right margin,

$$[-pI + \mu(\nabla \vec{u} + (\nabla \vec{u})^T)] \cdot \hat{\vec{n}} + \hat{\vec{p}}_{ref} \cdot \hat{\vec{n}} = 0$$

. p_{ref} represents the reference pressure 1.01×10^5 Pa. For the rest of the interface,

$$\vec{u} \cdot \hat{\vec{n}} = 0$$

. It means that there is no viscous force.

Similarly, we care the temperature field of the system. An hour later, the temperature field is characterized by the following figure, from which we conclude that the asymmetry of the temperature field is caused by the exact wind direction from left to right.

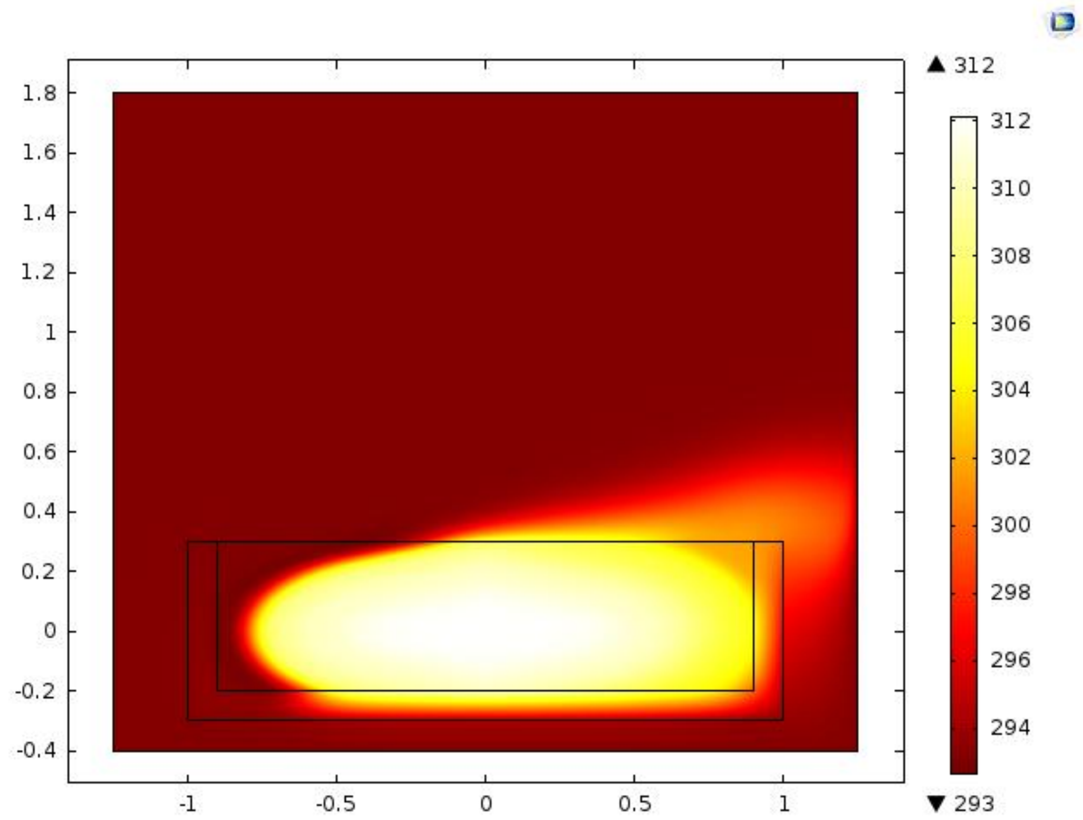

Figure 4.2.2-1 The temperature field 1 hours later

Obviously, the system will also get to a steady state finally. Besides providing the temperature field after 100 hours to verify the steady state, we calculate the distribution of flow field.

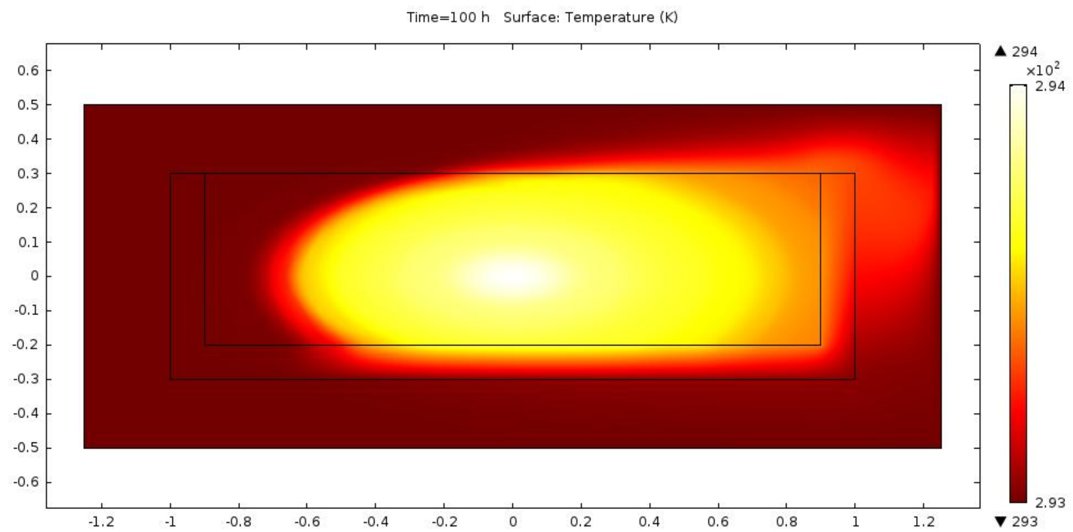

Figure 4.2.2-2 The temperature field 100 hours later

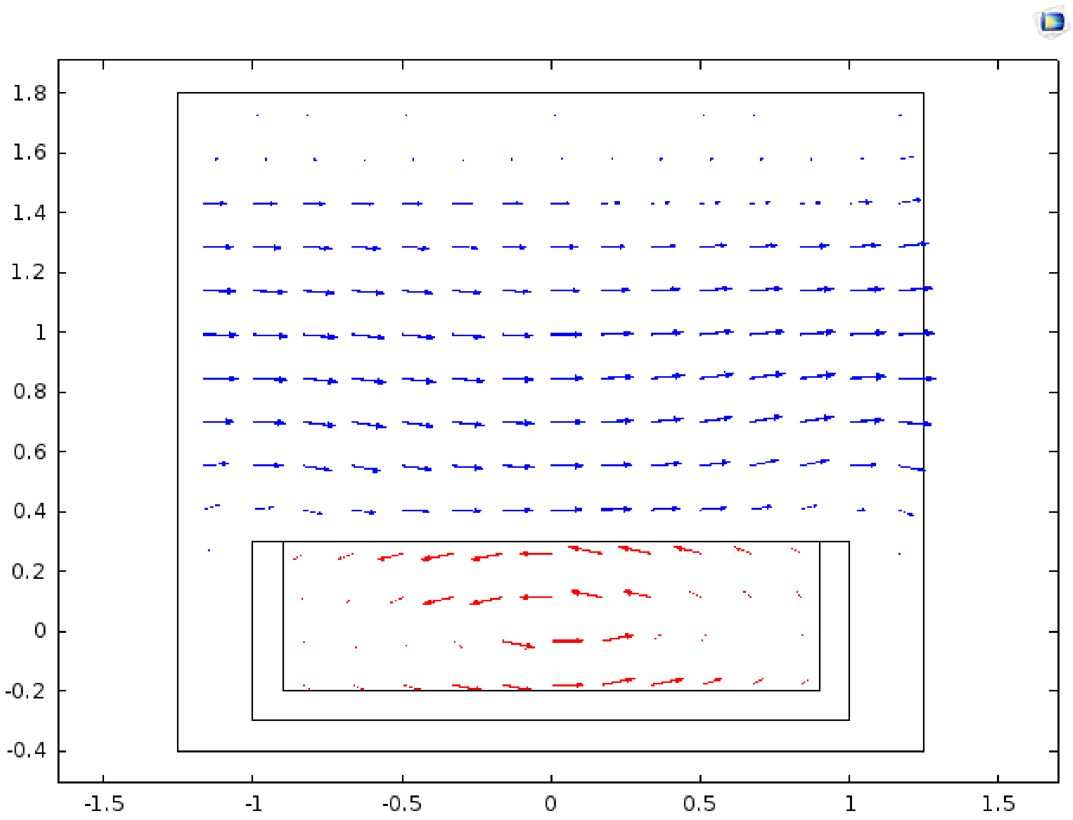

Figure 4.2.2-3 The distribution of flow field

In order to make the illustration more complete and convictive, veracious relationship between time and the average of the water temperature is shown below. One of the most remarkable things is that the curves are different to some extent due to the different ways the water flow stirs.

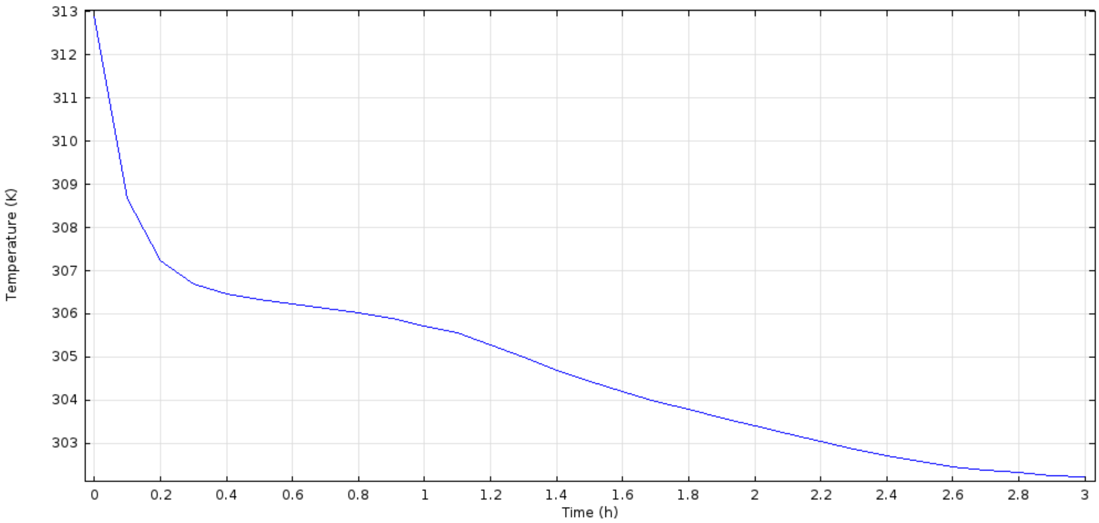

Figure 4.2.2-4 The relationship between time and the average of the water temperature

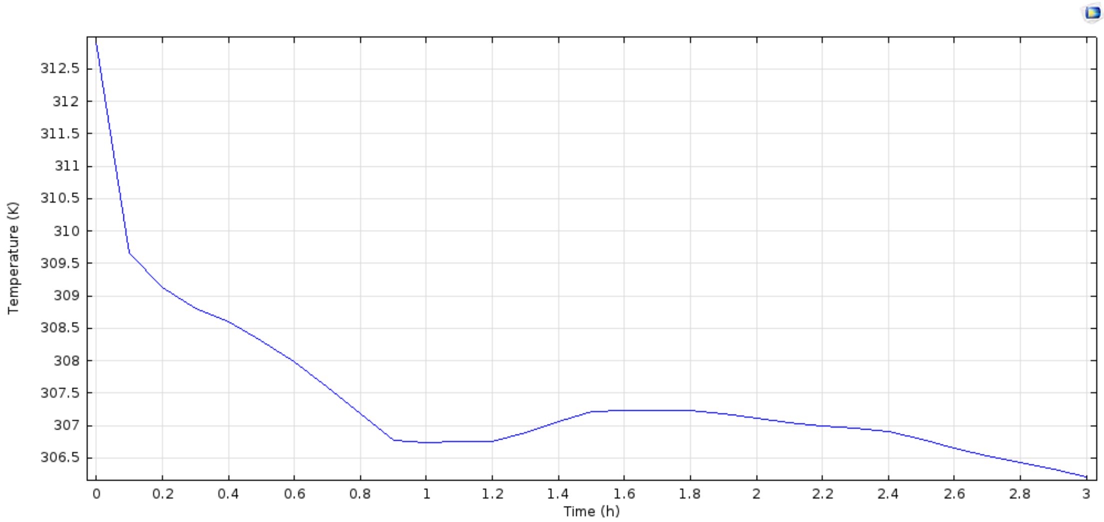

Figure 4.2.2-5 The relationship between time and the average of the water temperature

Anyway, this model does better than the first one because the result——the water temperature declines at first quickly and then slowly——is consistent with our common sense.

4.2.3 Remarks

Together with the simplest model, by confirming the law of natural cooling of the water, this model not only verifies the validity of our models but also lay a foundation for the following models.

4.3 The third model

4.3.1 Assumptions and relative theories

By taking a constant trickle of hot water from the faucet and consequently the water wasted into account, this model is more close to the actual situation, though the bather is still left out of the consideration.

According to the assumptions we make above, the eddy current field we set in the second model is wholly unbefitting and therefore eliminated.

The useful formulas include the heat equation

$$\rho c_p \frac{\partial T}{\partial t} + \rho c_p \vec{u} \cdot \nabla T = \nabla \cdot (k \nabla T)$$

and fluid equation

$$\begin{cases} \nabla \cdot [-pI + \mu(\nabla \vec{u} + (\nabla \vec{u})^T)] + F = 0 \\ \nabla \cdot \vec{u} = 0 \end{cases}$$

4.3.2 Model theory

For the bathtub,

$$\rho c_p \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T)$$

For the water and the air, they have a set of same equations

$$\begin{cases} \rho c_p \frac{\partial T}{\partial t} + \rho c_p \vec{u} \cdot \nabla T = \nabla \cdot (k \nabla T) \\ \nabla \cdot [-pI + \mu(\nabla \vec{u} + (\nabla \vec{u})^T)] + F = 0 \\ \nabla \cdot \vec{u} = 0 \end{cases}$$

. The temperature of the hot water from the faucet is defined as 318K at first.

To solve these equations, we are asked to educe a stable flow field according to the fluid equations and then substitute the velocity $\vec{u}$, a constant, into the heat equation. Thus the initial conditions of the fluid equations are unnecessary. Here are six boundary conditions of the fluid equations.

For the air flowing from the left margin,

$$\nabla \cdot [-pI + \mu(\nabla \vec{u} + (\nabla \vec{u})^T)] + f_{flow} = 0$$

. Determined by the viscidity of the fluid, the initial velocity the air need, the pressure and the velocity of the fluid, f_{flow} represents the force required per unit volume when the air is forced to the fluid. For the water flowing from the water inlet, semblable boundary condition is proposed.

For the air flowing out of the system through the right margin,

$$[-pI + \mu(\nabla \vec{u} + (\nabla \vec{u})^T)] \cdot \hat{\vec{n}} + \hat{\vec{p}}_{ref} \cdot \hat{\vec{n}} = 0$$

. p_{ref} represents the reference pressure 1.01×10^5 Pa. For the water flowing away through the overflow drain, semblable boundary condition is proposed.

For the interface between the air and the water,

$$\vec{u} \cdot \hat{\vec{n}} = 0$$

. It means that there is no viscous force.

For the interface between the air and the bathtub and the water and the bathtub,

$$\vec{u} = 0$$

.

As for the heat equation, the interface between the fluids and the tub has no boundary conditions as there are three parameters on one side while only one on the other.

Then we present the result of our simulation.

The temperature field of the system half an hour later is shown below.

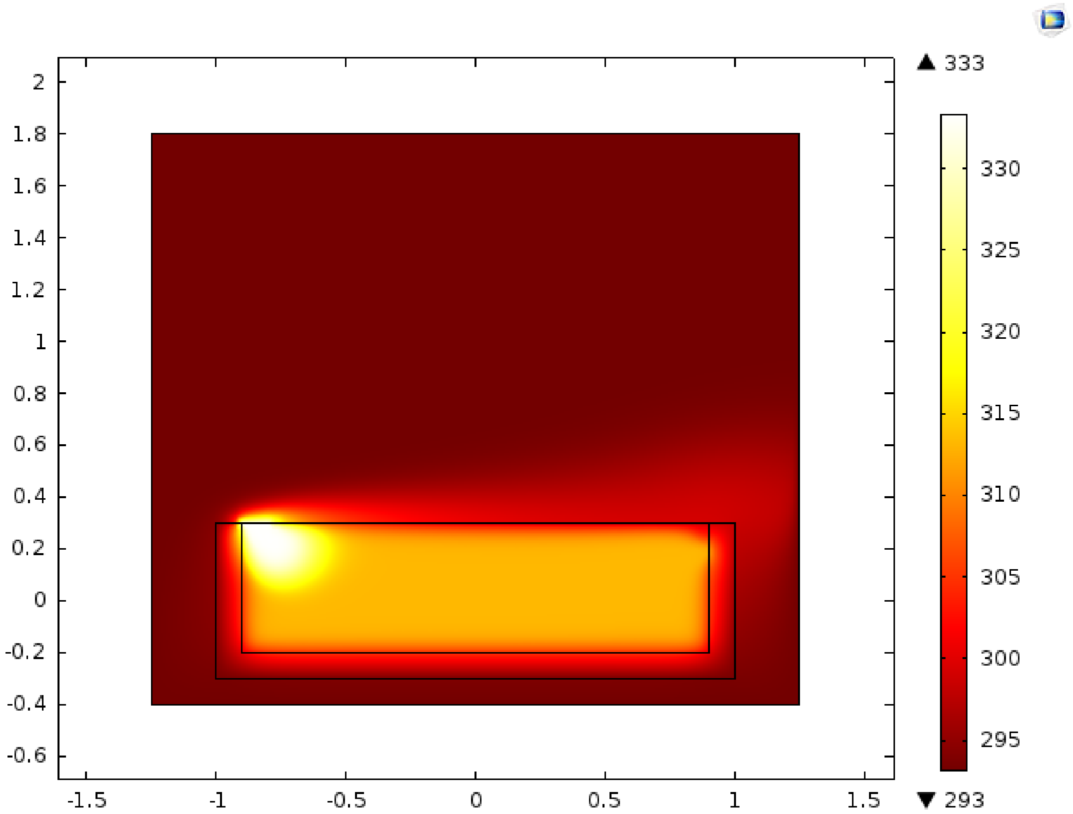

Figure 4.3.2-1 The temperature field half an hour later

Now that the hot water from the faucet is defined as 318K, the velocity of flow is the most meaningful variable. The following figure shows us certain relationships between time and the average of the water temperature when the flow velocity of the hot water are respectively $1\times e^{-4}$m/s, $2\times e^{-4}$m/s, $3\times e^{-4}$m/s, $4\times e^{-4}$m/s, …, $9\times e^{-4}$m/s and $10\times e^{-4}$m/s. The black curve shows the relationship when the flow velocity is $7\times e^{-4}$m/s.

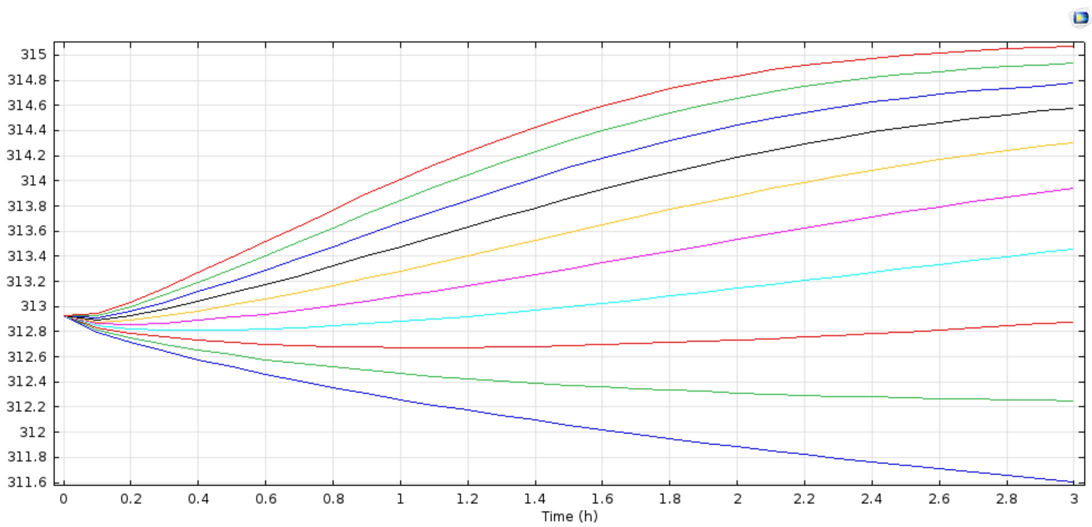

Figure 4.3.2-2 The relationships between time and the average of the water temperature

The water temperature can rise up when the flow velocity of the hot water (318K) is $4\times e^{-4}$m/s or faster.

To make the illustration more complete and convictive, we also elicit the relationships between time and the average of the water temperature when the flow velocity of the hot water is constant and the temperatures of the hot water added are respectively 311K, 312K, 313K, ..., 320K and 321K. The result is shown below. The black curve shows the relationship when the temperature of the hot water is 44K.

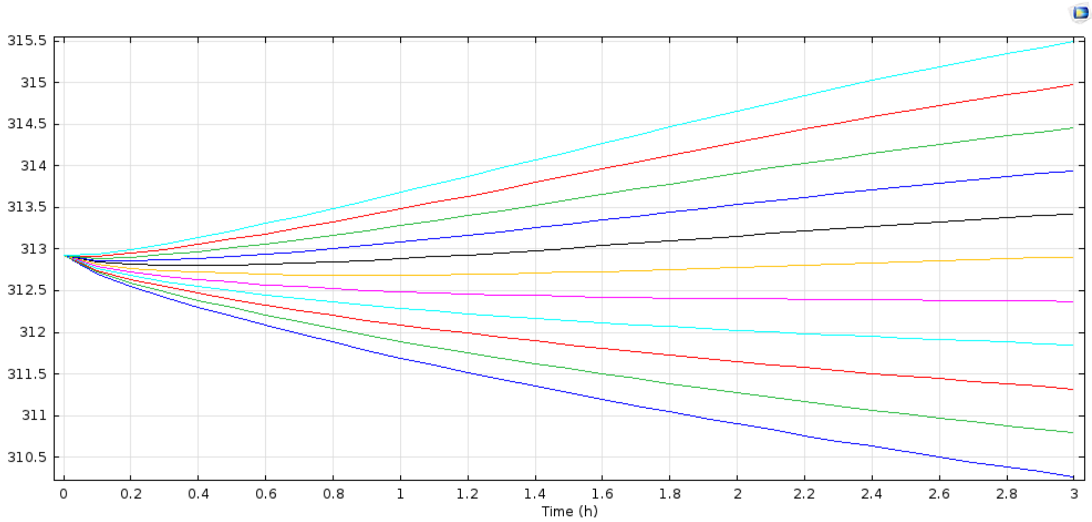

Figure 4.3.2-3 The relationships between time and the average of the water temperature

The water temperature can rise up when the temperature of the hot water added is 317K or higher.

Furthermore, we calculate the distribution of flow field.

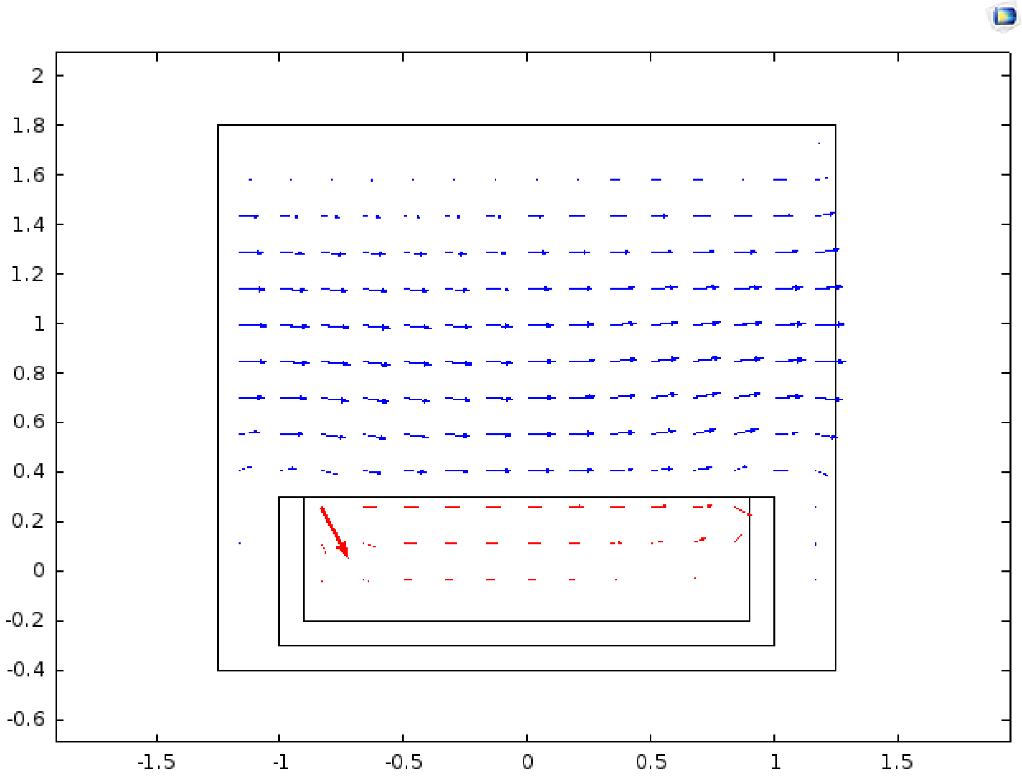

Figure 4.3.2-4 The distribution of flow field

4.4 The model that considers the person

4.4.1 Assumptions and relative theories

On the basis of the third model, this one almost meets the demand by taking the bather into account. The other assumptions we have made and the relative theories we used in the third model are still appropriate for this model.

4.4.2 Model theory

Compared to the third model, the only difference is that the boundary conditions relative to the bather should be appended. Given that making him or her motionless is somewhat tolerable, the boundary condition of the fluid equation is

$$\vec{u} = 0$$

and the boundary condition of the heat equation is the Dirichlet boundary condition

$$T = T_{human} = 310K$$

. Similarly, we do the same thing as we did in the third model and get the following figures.

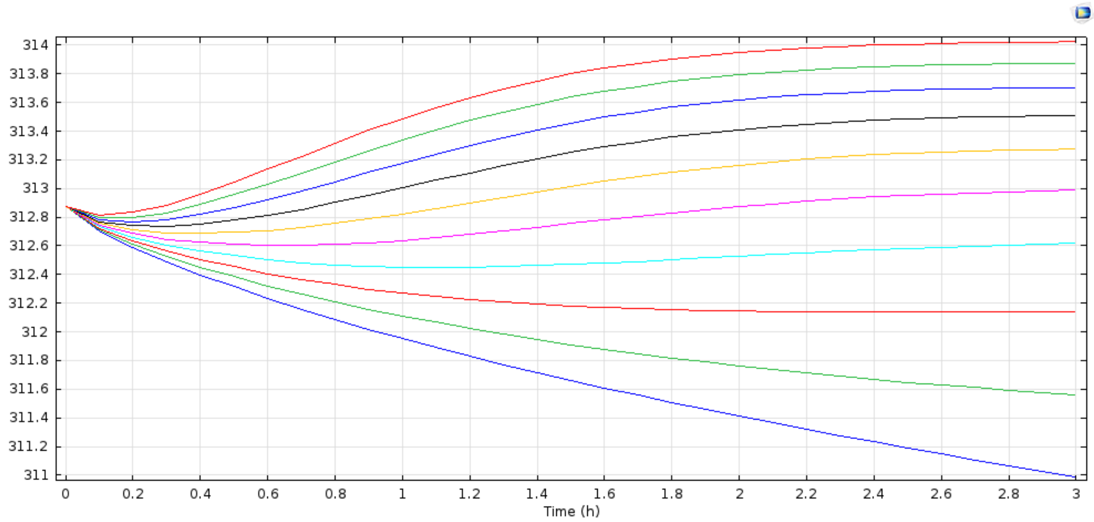

Figure 4.4.2-1 The relationships between time and the average of the water temperature

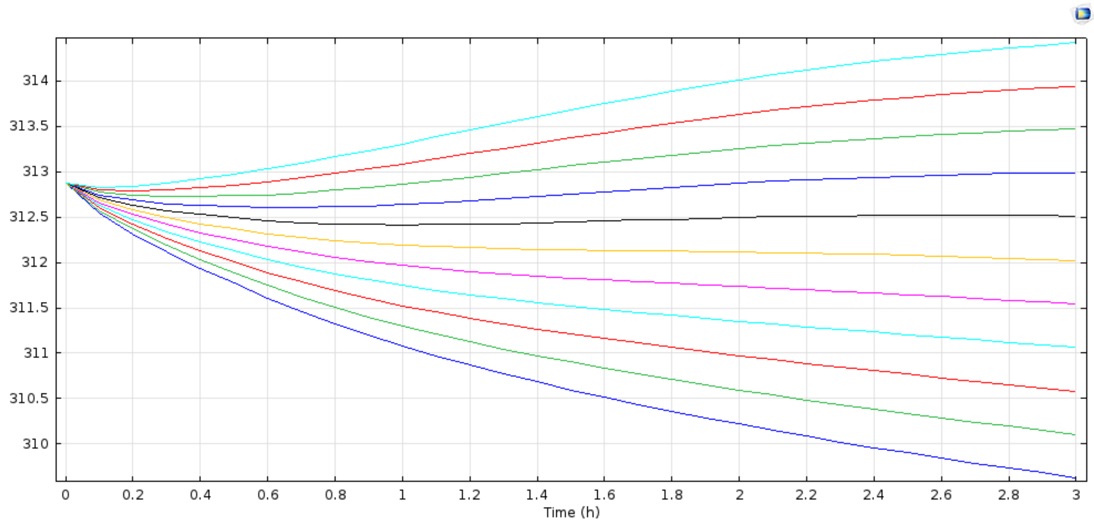

Figure 4.4.2-2 The relationships between time and the average of the water temperature

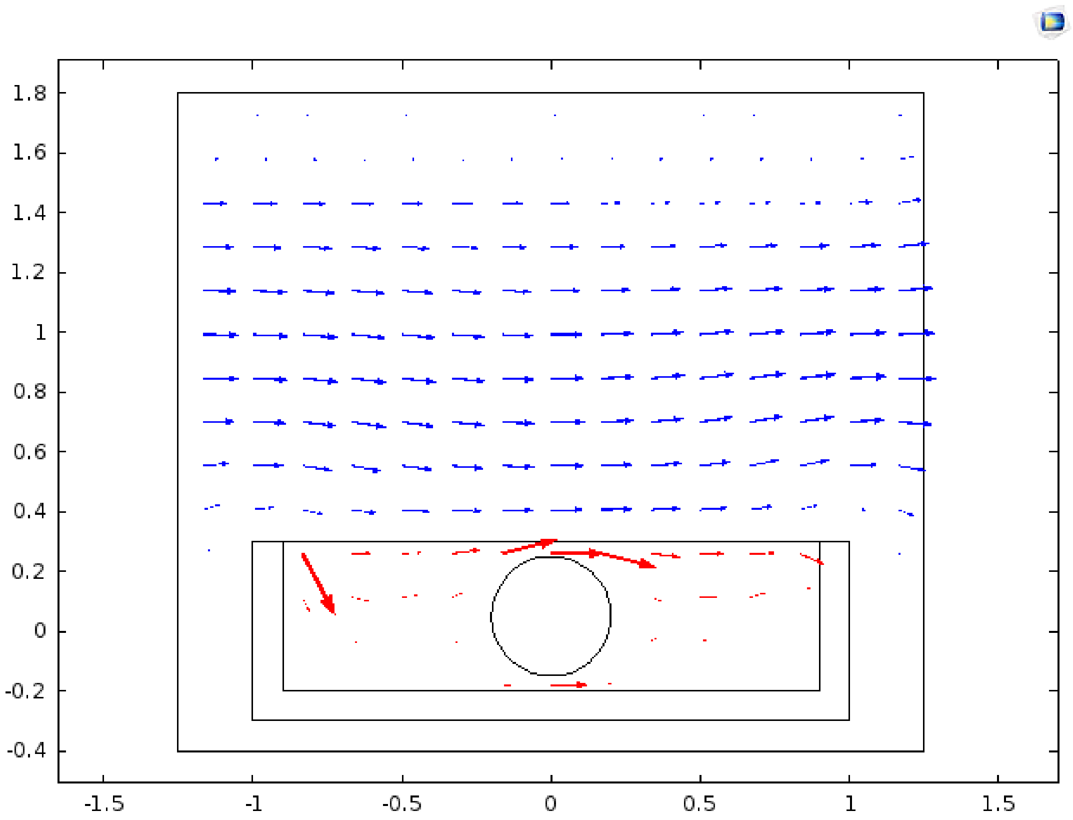

Figure 4.4.2-3 The distribution of flow field

4.4.3 Remarks

Compared with the result of the third model, we can reach the following conclusions:

a. The bather really makes a difference to the water temperature.

b. The distribution of flow field and the distribution of temperature field are influenced observably by the bather.

c. The tendence of declining has trifling changes.

d. The bather makes great contribution to the stabilization of the water temperature.

e. When the flow velocity of the hot water (318K) is $4\times e^{-4}$m/s, the water temperature finally will almost stop at a comfortable temperature——313K. So (318K, $4\times e^{-4}$m/s) can be seen as a solution of this problem.

4.5 The last model

4.5.1 Assumptions and relative theories

On the basis of the forth model, this one tries to make an explanation about the situation where a bubble bath is added to the system. The other assumptions we have made and the relative theories we used in the forth model are still appropriate for this model. What can we do with the bubbles?

4.5.2 Model theory

The heat conductivity coefficient of the bubble can be regarded as the heat conductivity coefficient when the air is static because the air in the bubble is not flowable. Thus the equation is

$$\rho c_p \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T)$$

, in which only k is different from what we defined before.

The result is displayed as follow-up.

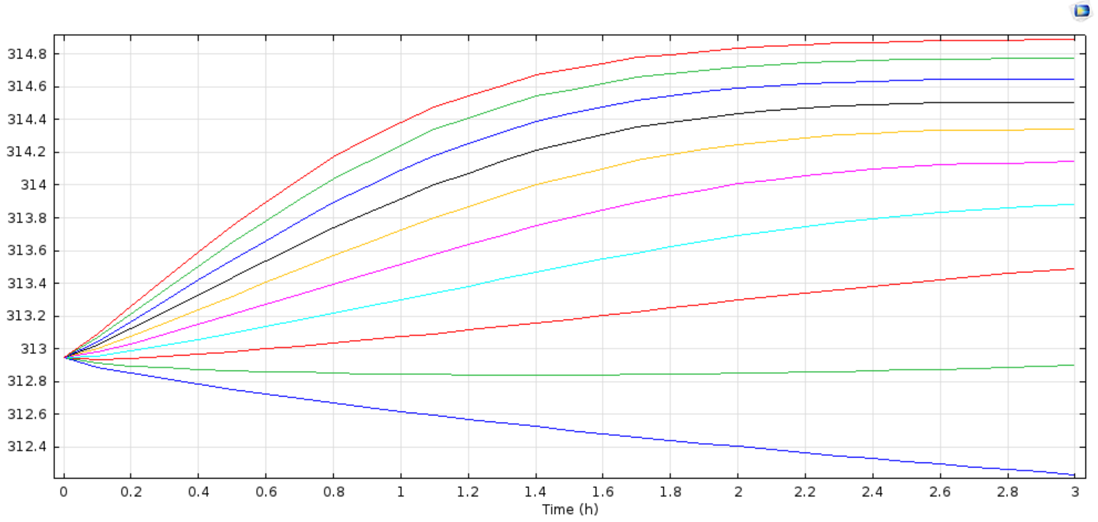

Figure 4.5.2-1 The relationships between time and the average of the water temperature

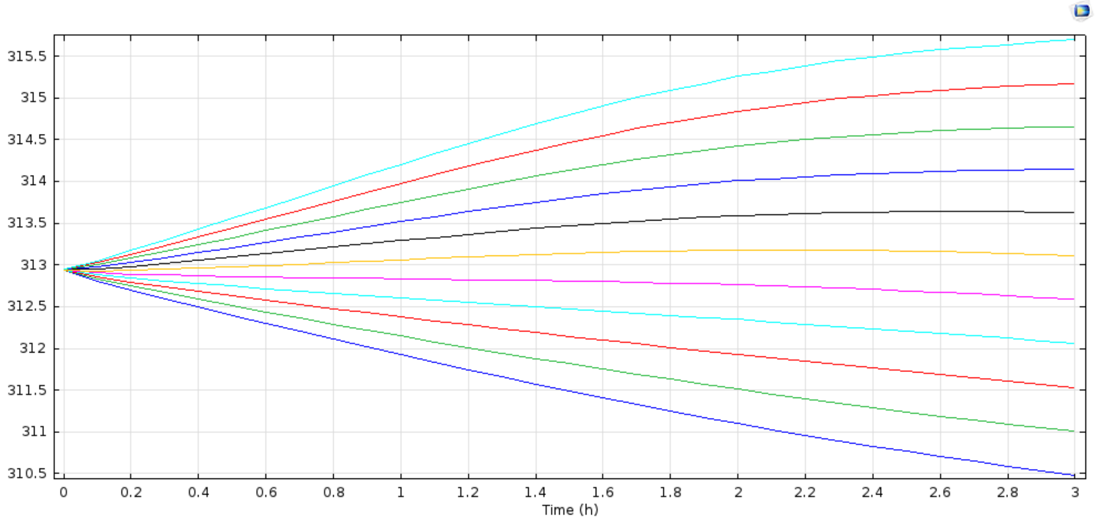

Figure 4.5.2-2 The relationships between time and the average of the water temperature

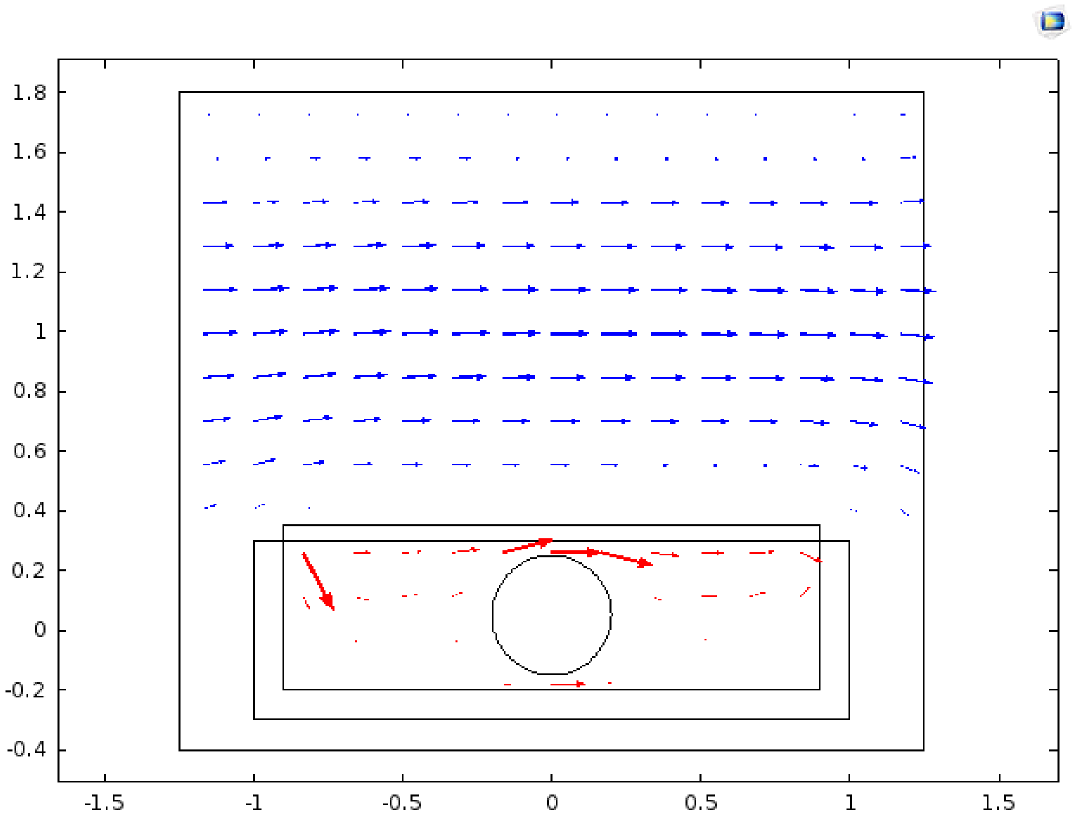

Figure 4.5.2-3 The distribution of flow field

In addition, different from the previous models, this model provides a figure which tells us the temperature field of the system two hours later.

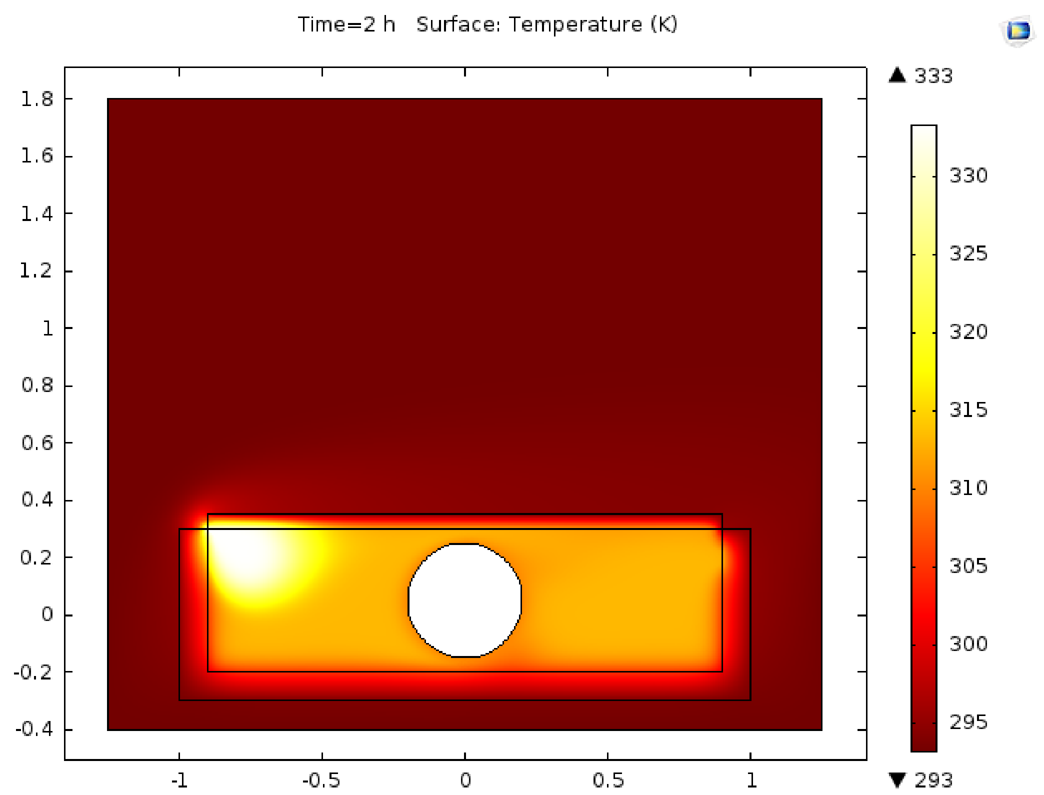

Figure 4.5.2-4 The temperature field two hours later

4.5.3 Remarks

Compared with the forth model, we know that bubbles do better than water in terms of both heat preservation and thermal insulation.

4.6. Discussions

a. The heat exchange coefficient between the water and the air, the water and the bathtub and the air and the bathtub are serviceable only when the water and the air are static. That is why only the simplest model used them.

b. The heat exchange between the water and the air is easier than we suggest it to be since evaporation is non-ignorable. The existence of decalescence requires us to define the system as a non-conservative one, just the opposite of what we did. Therefore, errors is inevitable.

c. In this model, the bather is regarded as a upturned cylinder. Since we use the boundary condition of the heat equation to determine the bather's influence on the system, his or her shape or volume is not as important as we think. However, the temperature of the person is meaningful because the boundary condition is

$$T=T_{human}$$

. Hence, the solution depends on the temperature of the person to a great extent.

d. The motions made by the person in the bathtub, by accelerating the evaporation and the water flow, are also crucial. This situation can be simplified as a model we often see in our daily lives——we stir hot milk with a fork in order to finish our breakfast more quickly. Therefore the motions made by the person is a determinant. Considering that bathers generally make low-frequency movements when they take a bath, the motions they make can also be ignored.

e. The bathtub is a cuboid in this model and its volume V has been used many times in our deducing. Thus, this model depends tremendously on its shape and the size of it.

A letter for users of the bathtub

Dear readers,

It is my great honor to serve the explanations of the bathtub here. As you can see, the bathtub is a simplest one with an overflow drain. Lacking a secondary heating system, the water in the bathtub needs to be reheated by adding hot water continuously, or the water temperature would soon be cooled down. Excess water would escapes through the overflow water, which can spare your attention off dealing with it. In the next paragraph, I would like to introduce a strategy of using water to help you enjoy a comfortable bath without wasting too much water.

First, bath in a room as small as possible and try to avoid the flow of air. Flowing air will take away your heat undeniably, as you can imagine. Using a bubble bath additive can not only cover your body perfectly but also block the heat exchange between water and air, which helps you keep the water temperature in a better way. Make the hot water flow from one side and the excess water spills into the other side if you can, for if you do so, it is the coldest water that spills out. Water in your bathtub is not at the same temperature, and the water which is furthest from the hot water newly in is obviously the coldest.

Water temperature distribution do assist in the bathtub, different boundary conditions also suggest that. So getting an even temperature throughout the bathtub is very difficult, unless you inconstantly moving as a way of stirring water.

If you can have the hot water at a higher temperature, it surely will save the water, controlling the speed of hot water to a scope of which the temperature of the water is acceptable for you.

Now enjoy your nice day by taking a relaxing bath!

Reference

[1]Zerkle, R. D. (1964). Sieder, E. N., & Tate, G. E. (1936). Laminar-flow heat transfer and pressure drop in tubes with liquid solidification.
[2]Heat transfer and pressure drop of liquids in tubes. Industrial & Engineering Chem istry, 28(12), 1429-1435.
[3]Webb, R. L., & Kim, N. H. (1994). Principl of enhanced heat transfer. Taylor Franci s: New York, NY, USA.