Team No. 507

Problem B

Extraterrestrial Life

Abstract

In this problem, we are required to analysis on a four-legged animal on the equator of an earth-like planet. To solve the problem, we set up a model based on invariable foundations of organism – the fixed scale of primary biochemical molecules and the common pattern of locomotion of four-legged animals. The result predicts the maximum speed of locomotion and size of the animal on the earth-like planet with twice the gravity of earth and temperature of 250K. After calculating our model, we give our conclusion: animals on that planet tends to have a mass of 80-600kg and maximum speed of locomotion of 100m/s. The strengths and weaknesses of our model are also discussed at the end of the article.

1 Introduction

Recently, scientist have found many earth-like planet orbiting around other stars – some of them are believed to satisfy the conditions of evolving life such as a relatively stable celestial environment and existence of liquid water.

We are interested in the characteristics of a four-legged animal if such a life form exists on a planet slightly different from earth. We consider a specific case that the planet has a mass which is eight times that of earth and a radius which twice that of earth. Also, the average temperature of the planet is 250K which is much lower than earth.

To specify the question of what the animals look like, we choose some characteristic physical quantities to judge the animals' appearance. First of all, we choose the average mass of the animal to indicate the size of the animal. Mass is one of the most credible quantities to indicate the level of metabolism and biological activity and it is a fundamental quantity that constrain the size and function of the organs, the efficiency of circulatory system and behavior of animals. Whereas other quantities relating to size, such as shoulder height and whole body length, are not as objective as mass. Secondly, we analysis the intake of oxygen which indicates the rate of metabolism of the creature and helps us further study the behaviorism the creature. Finally, we are specifically interested in the speed limit of the creature.

To roughly estimate the scale of body mass of the four-legged animal qualitatively, we find out upper and lower limits of body mass. The body mass can't be too small, for too small a size means the animal cannot produce enough heat equivalent to the heat that losses. It's because volume diminish faster than the surface area when the size of the animal decreases. Ratio of volume to surface area is crucial to the equilibrium of heat, in that volume and surface area is proportional to heat production and heat loss respectively. On the other hand, the body mass cannot be too large, for the skeleton of the animal cannot bear the heavy body of the animal as well as afford the acceleration and deceleration during locomotion. For example, the mass of *Baluchitherium* which is 80 tons is almost the ultimate of body mass of terrestrial animal.

The activity of metabolism of the creature as well as the consumption rate of oxygen is constrained by the body mass, thus limiting the capacity of locomotion. The rate of chemical reaction inside the organism is constant, for the structure of biological molecules and the mechanism of organism do not vary much from planet to planet which will be proved later in the article.

In a word, the body mass, the consumption rate of oxygen and the capacity of locomotion have a range which is determined by the innate mechanism of organism and the environment.

2 Model

2.1 Assumptions

In this problem, we assume the earth-like planet has a chemical environment quite resembles the earth which is rich in carbon, oxygen, nitrogen, phosphorus and sulfur which are the six key elements for life on earth. Abundance of these key elements lays a solid foundation for us to believe that composition of the animals on this planet is also C-O-N-P-S and this enables the life form on this planet shares quite similar mechanism of that on the earth. Therefore, this drives us to assume that the animals on this planet also use respiration process to produce energy, i.e. chemical energy released in the oxidation reaction is utilized and the oxygen plays an important role in the supply of the energy.

We are limited to study the four-legged animal and nearly all of the four-legged animal on earth belongs to Mammalia which are all homothermal animals. Mammals exchange oxygen and other nutrients in water phase (blood) unlike some arthropods exchange oxygen in gas phase through Hence, we assume that the four-legged animals on that planet is also warm blooded and use vascular system to undertake the exchange of material and energy.

2.2 Model Overview

Our model can be divided into four parts.

- A tunnel model is established to resemble the network that animals use to transport oxygen to all parts of its body. Heat production depends on oxygen consumption and the rate of oxygen consumption is confined by the mechanism of the tunnels.
- Equilibrium of heat production and heat loss and the animal's capacity of regulation confines the lower limit of body mass.
- A mechanic model based on the common pattern of four legged animals' locomotion gives the maximum force the animal exerts on the bones of the limbs, thus confining the upper limits of the body mass.

- The above mechanic model which gives the function of maximum speed of locomotion with respect to maximum oxygen consumption, combining with outcome of the tunnel model, can give the ultimate speed of the animal. Meanwhile, the mechanic model illustrates a possible and visualized graphics of the moving style of extraterrestrial life.

2.3 Model in Detail

2.3.1 Tunnel Model

Tunnels are needed for living animals to transport oxygen, such as aorta, arteries, arterioles, and capillaries, to supply all parts of the organism. And they can be assumed as a fractal system, which should be finitely self-similar for the constraint set by the size of capillaries.[1] There are three unifying principles or assumptions: First, a space filling fractal-like branching pattern is required. Second, terminals of branches are size-invariant because it's constrained by the size of atoms and molecules. And third, the energy required to distribute resources is minimized. By Geoffrey B. West's *WBE model*[2], a highly approximate equation as follow could be derived.

$$R = (M_b/M_0)^{\frac{3}{4}} \qquad (1)$$

Where R means the rate of metabolism of the mammal, M_b is the mass of its body and M_0 is a normalization scale for M_0.
On the basis of *WBE model*, we could say that the rate of metabolism is independent of the temperature of environment and the acceleration of gravity.
Then we find some data about the metabolic rate of different mammals

	Monotreme	Marsupial	Eutherian
Body temperature(˚C)	30	35.5	38
Metabolic rate(W)	$1.65M_b^{0.75}$	$2.36M_b^{0.737}$	$3.35M_b^{0.75}$
Metabolic rate, recalculated to 38(˚C)	$2.99M_b^{0.75}$	$2.97M_b^{0.737}$	$3.35M_b^{0.75}$

Table 1. Energy metabolic rates of higher vertebrates, recalculated to a uniform body temperature of 38 (body mass, Mb, in kg, metabolic rate in watts) (from Dawson and Hulbert, 1970).

For the animal on the earth-like planet, we use the equation

$$R = 3.34{M_b}^{3/4} \qquad (2)$$

As we have assumed the animal as a eutherian to calculate its rate of metabolism.
We've also build another model which gives the similar result.
The animal can be regarded as the combination of several columns. For each infinitesimal unit we can do approximate calculations on the rate of energy it consumes. We use the column as the infinitesimal unit.
We have,
The work it does in a movement of $\Delta\ell$, $W \propto \sigma A \Delta \ell$ (3)
The power, $P \propto \sigma A \Delta \ell/\Delta t$ (4)
σ and $\Delta \ell/\Delta t$ can be considered independent of size,so $P \propto A \propto d^2$ (5)
On the other hand, the mass of the body, $M_b \propto \ell\, d^2$ (6)
The mathematician Greenhill showed that when the force due to weight is distributed over the total extent of the column, the critical length becomes:

$$\ell_{cr} = 0.792 \left|\frac{E}{\rho}\right|^{1/3} d^{2/3} \qquad (7)$$

Where E is the elastic modulus of the material, ρ is the density of the material. It's obvious that $\ell^3 \propto r^2$ (8)
Combining the (6) and (8), we have $d \propto {M_b}^{\frac{3}{8}}$ (9)

At last, with the equation (5), the rate of work $P \propto {M_b}^{\frac{3}{4}}$ (10)

This is of the same form of (2).

2.3.2 Equilibrium of heat

On Earth, mammals have to maintain their body temperature relatively constant to guarantee the basic biological functions of enzymes. So their body temperature is independent of the environmental temperature. It is obvious that all homothermal mammals have to maintain a balance between the rate of heat production and heat loss. As a result, the rate of metabolism is surely higher than the rate of heat loss to insure the stability of core temperature, for part of metabolism is utilized for basic activity of the animal.
Here are some mammal's core temperature:

Body mass(kg)	Number of species	Body temperature(℃)	
		Rang	Mean
0.001-0.01	2	37.8-38.0	37.9
0.01-0.1	11	35.8-40.4	37.8

0.1-1.0	12	35.8-39.5	37.8
1.0-10	17	36.4-39.5	38.0
10-100	8	36.0-39.5	37.9
100-1000	6	36.4-39.5	37.8
1000-10000	2	35.9-36.1	36.0
10000-100000	4	36.5-37.5	37.1

Table 2. Body temperatures of eutherian mammals arranged according to body size (Morrison and Ryser, 1952)

As we can see, core temperature of mammals is stable around 38℃ and independent of the size - namely their body mass. The table indicates that suitable and optimal temperature for the mammals is about 38℃. The animal on the earth-like planet, as assumed above, have the same biochemical reactions as mammals on earth. So we could can say that the core temperature of animals on that earth-like planet is also about 38℃. Thus, T_b=38℃.

However, heat loss is size-dependent, as larger mammals are easier to lose heat for they have larger surface area. So the consideration of heat conduction is needed. We assume the total conductance of the animal is C. According to Newton cooling law, we have

$$H = C(T_b - T_0) \qquad (11)$$

Where H is the rate of heat loss, T_b is temperature of body and T_0=250K is temperature of environment.

It is not hard to know that the thermal conductance C can be affected by the surface area exposed in the air, subcutaneous fat thickness, etc. It's not easy to get a specific value even for a single animal, because the conductance changes a lot when it rolls or stretches. Animals tend to roll themselves when it is cold, whereas they are stretched in hot days. However the efforts to minimize the surface exposed to the air to decrease the loss of heat is limited, so we have the minimized conductance at a low critical temperature. And at this low critical temperature, the only way to maintain stable core temperature is to product more heat. The following conductance we care about is the minimized conductance.

For the convenience to compare the conductance and the rate of heat production (metabolism rate), we express both in relation to body size (M_b). That is to the compare the C* (conductance per unit body mass) and H*(heat loss per unit body mass).

A considerable amount of data is accumulated by *Herreid and Kessel* (1967) that thermal conductance in relation to body size for mammals and birds, is given by the equation

$$C^* = 30{M_b}^{-0.5} \qquad (12)$$

where C* is specific thermal conductance[ml O_2 (kg·hr·℃)$^{-1}$]and Mb is body mass(kg)]. And the coefficient can readily be recalculated to other units by equating 1 liter of oxygen per hour with 5.579W.

Then, we can derive from equation (2) and (11) that：

$$R^* = 3.34{M_b}^{-\frac{1}{4}} \qquad (13)$$

$$H^* = C^*(T_b - T_0) \qquad (14)$$

As is stated above, we can get R*>H*. With equations (12), (13), (14), we can work out that $M_b > 81.7kg$, ie. $M_{b\,min} = 81.7kg$, which is the lower limit of the body mass of the animal.

2.3.3 Upper Limit of body mass

First, let's take a look on the situation that the animal stand still. We consider the four legs of the animal as columns which support the weight of the animal. So we have

$$M_{b_{max}} g = 4P_{max}A \qquad (15)$$

Where $M_{b_{max}}$ is the upper limit of body mass, g is gravity, P_{max} is the maximum pressure the bone can bear and A is the cross section area of the bones.

Consider the mass of the bones of limbs,

$$M_{bone} = \rho AL \qquad (16)$$

Where M_{bone} is the mass of the bone of one limb, ρ is density of bones, and L is the length of the bones which is proportional to $M^{1/3}$.

Increase in the mass of bones results in loss in mobility of the animal, thus causing the animal hard to survive. So we assume the ratio of the mass of bones and body mass is a constant and does not change among different planets. Also we assume ρ, P_{max} are invariable quantities in different planets. We get

$$\text{constant} = \frac{M_{bone}}{M_{b_{max}}} \propto g{M_{b_{max}}}^{\frac{1}{3}} \quad \text{ie.} M_{b_{max}} \propto g^{-3} \qquad (17)$$

Numerically, comparing with the heaviest four-legged terrestrial animal on the earth which is *Loxodonta africana* weighing 4800kg, the largest body weight of the animal on the planet mentioned in the problem is probably 600kg.

2.3.4 Spring-mass model

To analyze the locomotion of the 'extraterrestrial' animal, we are supposed to construct a physical model of motion. In some scholars' research [3][4], the simple spring-mass model, which employs a single linear leg spring and a mass, accurately predict the mechanics of running gaits. The body's complex system of active muscles, tendons and ligaments therefore behaves much like the single linear spring employed in the models. We will begin our construction with the simple vertical motion, consider a 2-Dimensional oscillation afterwards and obtain some useful information finally.

- Assumptions and equivalences

To expand our discussion, there are some assumptions and equivalences we have to make:

1. The stiffness of the leg spring k is defined as the ratio of the peak force and the peak displacement:

$$\mathrm{k} = \frac{F}{\Delta y} \qquad (18)$$

2. The length of the leg L_0 spring in the model is defined as the distance from the point of ground contact to the center of mass.
3. The mass of animal is abstracted as to concentrate into the mass point.

- Hop occasion

To begin our construction, let us consider a simple oscillation system (Figure 1) which vibrates in vertical direction—the 'hop' condition. To simulate the occasion in which animal has just landed in the ground, we set the initial displacement y0=0, initial velocity: $\dot{y}_a$ which is downward. After simple analysis we can know that the motion can be divided into two phases: the contact phase (in which the string contacts to the ground) and the aerial phase (in which the oscillator leaves the ground). [4]

1. Contact phase

We can easily obtain the motion equation:

$$m\ddot{y} + ky = -mg \qquad (19)$$

The general solution is:

$$y = a\sin\omega t + b\cos\omega t - \frac{g}{\omega^2} \quad \left(\omega^2 = \frac{k}{m}\right) \qquad (20)$$

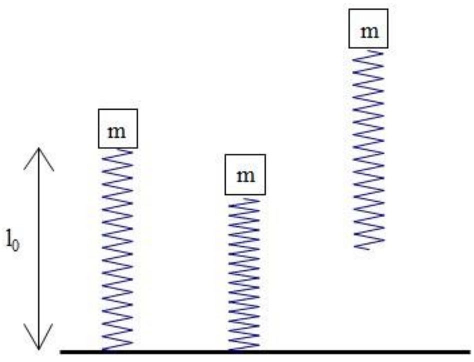

Figure 1 A simple oscillation system

The initial values are: ($\dot{y}_a$ is the absolute value of initial velocity)

$$y(t=0) = 0;\ \ \dot{y}(t=0) = -\dot{y}_a \qquad (21)$$

Therefore

$$b = \frac{g}{\omega^2};\quad a = -\frac{\dot{y}_a}{\omega} \qquad (22)$$

The result solution should be:

$$y = -\frac{\dot{y}_a}{\omega}\sin\omega t + \frac{g}{\omega^2}\cos\omega t - \frac{g}{\omega^2} \qquad (23)$$

Which is equivalent to:

$$y = -\dot{y}_a\left(\frac{m}{k}\right)^{\frac{1}{2}}\sin\omega t + g\left(\frac{m}{k}\right)\cos\omega t - g\left(\frac{m}{k}\right) \qquad (24)$$

Thus

$$F_{string} = -ky = \dot{y}_a(km)^{\frac{1}{2}}\sin\omega t - mg\cos\omega t + mg \qquad (25)$$

2. The contact time and the critical moment

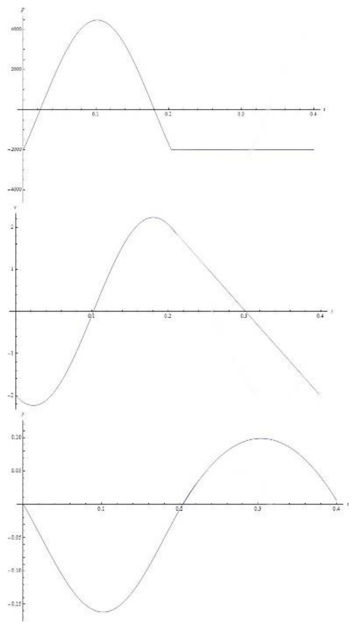
Figure 2

As is shown in Figure 2, in the middle of the total contact time, the string attain a maximal compression, and the velocity is exactly 0, in other words, $\dot{y}\left(t=\frac{t_c}{2}\right)=0$. (Where t_c is the total contact time), thus we get:

$$\dot{y}_a \cos\frac{\omega t_c}{2}+\frac{g}{\omega}\sin\frac{\omega t_c}{2}=0 \qquad (26)$$

Therefore:

$$\tan\frac{\omega t_c}{2}=\dot{y}_a\frac{g}{\omega} \iff t_c=\frac{2}{\omega}\tan^{-1}(\dot{y}_a\frac{g}{\omega}) \qquad (27)$$

So we get the expression of the contact time t_c.

When $t=t_c$, it's obvious that

$$\dot{y}(t=t_c)=\dot{y}_a \qquad (28)$$

3. Aerial phase

The motion equation is:

$$m\ddot{y}=mg \qquad (29)$$

Then we get: (notate ta: the aerial time)

$$\dot{y}_a=\frac{gt_a}{2};\quad t_a=\frac{2\dot{y}_a}{g} \qquad (30)$$

The total period:

$$T=t_a+t_c=\frac{2\dot{y}_a}{g}+\frac{2}{\omega}\tan^{-1}(\dot{y}_a\frac{g}{\omega}) \qquad (31)$$

Now we can plot the relationship between F, v, y and t, which could give us a visualized motion process during the hop.

- First-try of Gallop (Running) Model

We assume that the locomotion of the extraterrestrial animal appears like the 'gallop' (jump), just like most of the earth animal - horse, kangaroo, wolf and so on.

We still use the spring-mass model to describe the motion. As the Figure 3 shows, there are also two phase of motion - contact motion and aerial motion.

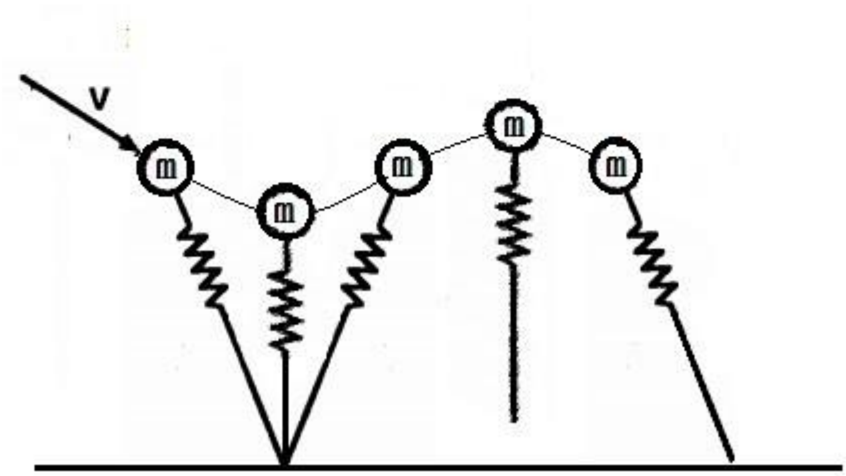

Figure 3 Two phase of motion

Firstly, the aerial motion is easy to do research on: it's an oblique projectile motion, where the horizontal velocity is a constant, and vertical motion is the same as the aerial of hop occasion.

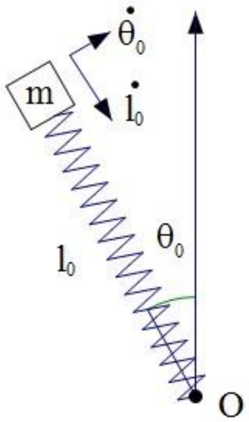

Figure 4In a polar coordinate

Then our main task is to research the contact phase, which is a 2-Dimensional Oscillation. We assume that the contact end of the spring will not move during a contact phase, and the mass point rotates around the contact point. The research is done in a polar coordinate. Set the origin on the contact point, the parameters—$l, \dot{l}, \theta, \dot{\theta}$ are shown in Figure 4. The motion equations are:

$$\begin{cases} -k(l-l_0) - mg\cos\theta = \ddot{l} - l\dot{\theta}^2 \\ mg\sin\theta = l\ddot{\theta} + 2\dot{l}\dot{\theta} \end{cases} \tag{32}$$

And the initial value:

$$l(t=0) = l_0;\ \dot{l}(t=0) = \dot{l}_0;\ \theta(t=0) = \theta_0;\ \dot{\theta}(t=0) = \dot{\theta}_0 \tag{33}$$

While this ODE group is too difficult to obtain an analytic solution! While the numerical solution is neither convenient for the analysis afterwards nor intuitive to illustrate the relationship that we want to know we will try to simplify the 2D model.

- Simplification for Gallop Model

1. Assumptions

We just need to simplify the contact phase. Now we have to make some new assumptions:

1) The horizontal speed v_x is almost constant during the contact phase. This assumption is proved by Farley and associates (1993) [3] in earth mammals. Now we extend this assumption to extraterrestrial life.
2) Let's introduce a new parameter: the effective vertical stiffness—k_{vert}, which is defined as the ratio of the peak vertical force and the peak vertical displacement[3]:

$$k_{vert} = \frac{F_{max}}{\Delta y} \qquad (34)$$

This parameter can be used equivalently to describe the stiffness in vertical direction.

While the real stiffness k_{real} is:

$$k_{real} = \frac{F_{max}}{\Delta L} \qquad (35)$$

ΔL, Δy are shown in the Figure 5.

2. Simplified Model

As is illustrates in Figure 5, 6, there are some geometrical relationship among the parameters.

Then we can get:

$$\Delta L = \Delta y + L_0(1 - \cos\theta) \qquad (36)$$

Thus

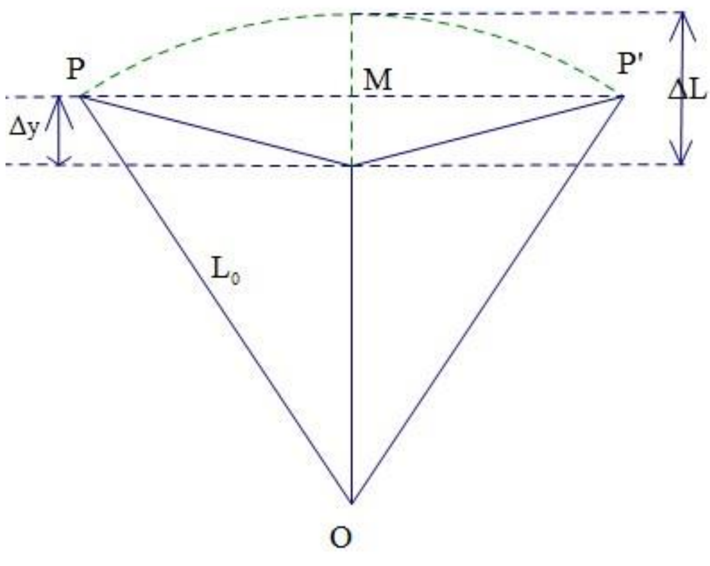

Figure 5

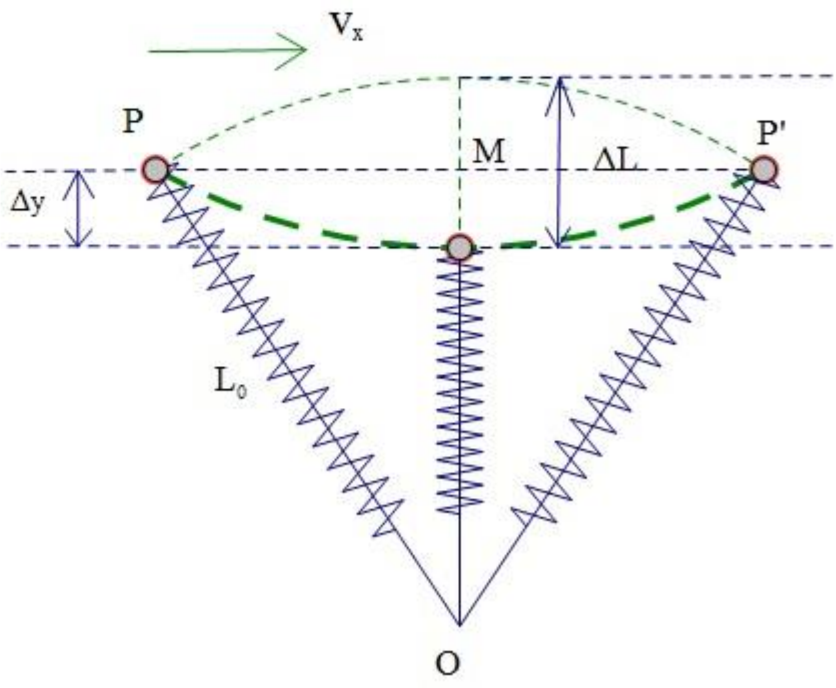

Figure 6

$$\mathrm{k}_{vert} = \frac{F_{max}}{\Delta y} = \frac{F_{max}}{\Delta L} \cdot \frac{\Delta L}{\Delta y} = \mathrm{k}_{\mathrm{real}} \cdot \frac{\Delta \mathrm{y} + \mathrm{L}_0(1-\cos\theta)}{\Delta y} \qquad (37)$$

$$\mathrm{k}_{vert} = \mathrm{k}_{\mathrm{real}} \cdot \left[1 + \frac{\mathrm{L}_0(1-\cos\theta)}{\Delta y}\right]$$

$$= \mathrm{k}_{\mathrm{real}} \cdot \left[\frac{\Delta \mathrm{L}}{\Delta L - \mathrm{L}_0(1-\cos\theta)}\right] \qquad (38)$$

On the other hand,

$$\mathrm{PP}' \approx \mathrm{v}_x t_c \qquad \text{(by assumption 1)} \qquad (39)$$

Therefore

$$\sin\theta = \frac{\mathrm{v}_x t_c}{2\mathrm{L}_0} \qquad (40)$$

3. Analogy to Hop Condition

As a result of assumptions and the model above, now we can simplify the vertical motion as a kind of hop motion, if only we replace the stiffness k by the effective vertical stiffness k_{vert}.

Now the conclusion of hop occasion can be similarly deduced:

$$\dot{\mathrm{x}}_0 = v_x, \dot{y}_0 = \dot{y}_a; \quad k = k_{vert}, \omega^2 = \frac{k_{vert}}{m} \qquad (41)$$

$$\begin{cases} m\ddot{y} + k_{vert}y = -mg \\ y(t=0) = 0; \dot{y}(t=0) = \dot{y}_a \end{cases} \qquad (42)$$

All above are the same equations. So the solution:

$$\mathrm{y} = -\frac{\dot{y}_a}{\omega}\sin\omega t + \frac{g}{\omega^2}\cos\omega t - \frac{g}{\omega^2} \qquad (43)$$

$$\mathrm{F}_{string} = -ky = \dot{y}_a(km)^{\frac{1}{2}}\sin\omega t - mg\cos\omega t + mg \qquad (44)$$

$$t_c = \frac{2}{\omega}\tan^{-1}(\dot{y}_a\frac{g}{\omega}) \qquad (45)$$

- Applications and discussion

1. The maximal force and its relationship with v, M

   By energy conservation, we can get:

$$\frac{1}{2}M{\dot{y}_0}^2 + Mgy = \frac{1}{2}k_{vert}y^2 \qquad (46)$$

Thus we get

$$\Delta\mathrm{y} = \frac{1}{k_{vert}}\left[Mg + \sqrt{M^2g^2 + M{\dot{y}_a}^2k_{vert}}\right] \qquad (47)$$

And then

$$\mathrm{F} = k_{vert}\cdot\Delta y = Mg + \sqrt{M^2g^2 + M{\dot{y}_a}^2k_{vert}} \qquad (48)$$

Now we attain an expression of maximal force and M, $\dot{y}_a$, and we know the force is proportional to

2. The rate of work and its relationship with v.

   From the dynamic equation of contact phase, we can easily get the expression of the rate of work:

$$\mathrm{P}_{spring} = \frac{0.5F^2f}{k_{real} * M} \qquad (49)$$

After deformation, we can get

$$\mathrm{P}_{spring} = \frac{\sqrt{k_{vert}}[\sqrt{M}g + \sqrt{Mg^2 + \dot{y_a}^2 k_{vert}}]}{4\pi k_{real} * M}$$

$$\approx \frac{\left[\sqrt{M}g + \sqrt{Mg^2 + \dot{y_a}^2 k_{real}}\right]}{4\pi\sqrt{k_{real}} * M} \qquad (50)$$

From the discussion in former pattern, we've got these information that the rate of work in metabolism is:

$$\mathrm{P}_{meta} = 3.34\mathrm{M}_b^{\frac{3}{4}} \qquad (51)$$

And researches on biology show the efficiency in transformation from chemical energy to useful energy is nearly 10%; and the output rate of work is a part of them. To estimate the extreme velocity, now we set:

$$\mathrm{P}_{spring} < 0.01P_{meta} \qquad (52)$$

And we assume the $\mathrm{M_b}$ is 100Kg; g=18.6m/s^2; $\mathrm{k_{real}}$ is set to be 50000N/m; then we got a rough answer:

$$\dot{y}_a < 100\mathrm{m/s} \qquad (53)$$

This rough number can testify our intuition that animal cannot run too fast to spill over the metabolism extreme however condition it is.

3 Results

After establishing two models concerning the metabolism rate and the pattern of locomotion respectively, we found out the relationship between physiological parameters of the animal and the parameters of the environment. Also, we find out specific value of the animals' physiological parameters on a specific planet, which, in detail, shows that a four-legged animal is tend to have a body mass between 80kg and 600kg and have an exhaust speed of no more than 100m/s, on a planet with twice the gravity on the earth and much lower temperature of the earth.

4 Discussion

4.1 Strengths

First, the equation we used to calculate the animal's rate of metabolism is very simple, which dependent on its size (mass). And it is strongly supported by a large amount of animals on earth, whether in equatorial regions or polar region and no matter what the size they are. What's more, the result we got is highly in agreement with another fancy model, which using fractal to prove the rate of metabolism increasing with three fourths power of body mass.

Second, the balance between the rate of heat loss and metabolism is considered to calculate the minimized mass of the animal on the earth-like planet, which is a requirement for all warm-blooded animals. After this, we could have a rough idea about the minimized size of the animal.

Third, the idea is profound and far-reaching to find the invariable and variable quantities that affect the physical process of the animal when the environment changes and when the animal are scaled, which reveals a ubiquitous rule of organism when transferred and scaled. And we've based our model on the homogenous part of biological mechanism, which is reasonable.

4.2 Weaknesses

First, the model is trivial and contains too much details. For example, the spring-mass model is too complicated and takes a lot into consideration. The outcome of the model shows a relationship of key parameters of animals and parameters of the environment, but it's hard for us to eliminate the impact of other irrelevant parameters.

Second, there are too much assumptions though most of them are reasonable and based on solid fact of nature. We assume the animals on that planet are quite similar to mammals on the earth, but in fact, it's hard to predict that.

Third, the model is just a rough case. For example, we can derive from our model that $M_{bone} \propto {M_b}^{-4/3}$, but experiment[5] shows that the exponent deviates from four thirds but a little smaller than that.

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