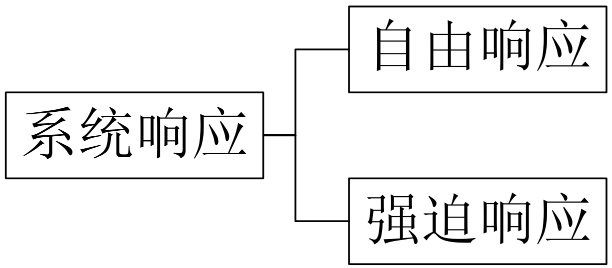
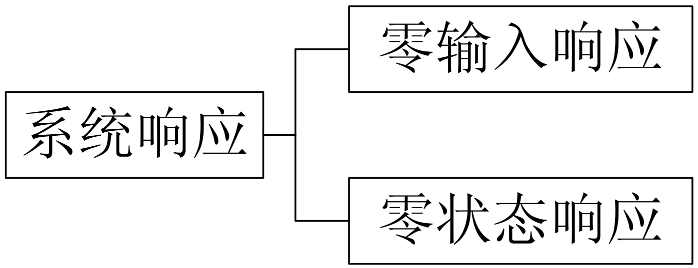
第二章

连续时间系统的时域分析

2.1 引言

- 时域分析方法的优点：
  - 直接研究系统的时间响应或时域特性
  - 直观、物理概念清楚，变换法的基础
- 方法：
  - 经典法
    - 高阶系统复杂
  - 卷积法
    - 线性时不变
    - 冲激响应之和





- 本章讨论的主要内容
  - 常系数微分方程的建立和求解-经典法
  - 零输入与零状态响应
  - 冲激响应和阶跃响应
  - 零状态响应的卷积法
- 时域分析方法是LT变换的基础
  - 便于理解、便于比较

2.2 常系数微分方程的建立和求解

- 常系数微分方程的建立
  - 物理模型转化为数学模型
  - LTI电网络（R，L，C）
  - 基尔霍夫电压及电流定律

KVL

$$V_R(t) = Ri_R(t)$$

$$V_c(t) = \frac{1}{c}\int_{-\infty}^{t} i_c d\tau$$

$$V_L(t) = L\frac{d}{dt}[i_L(t)]$$

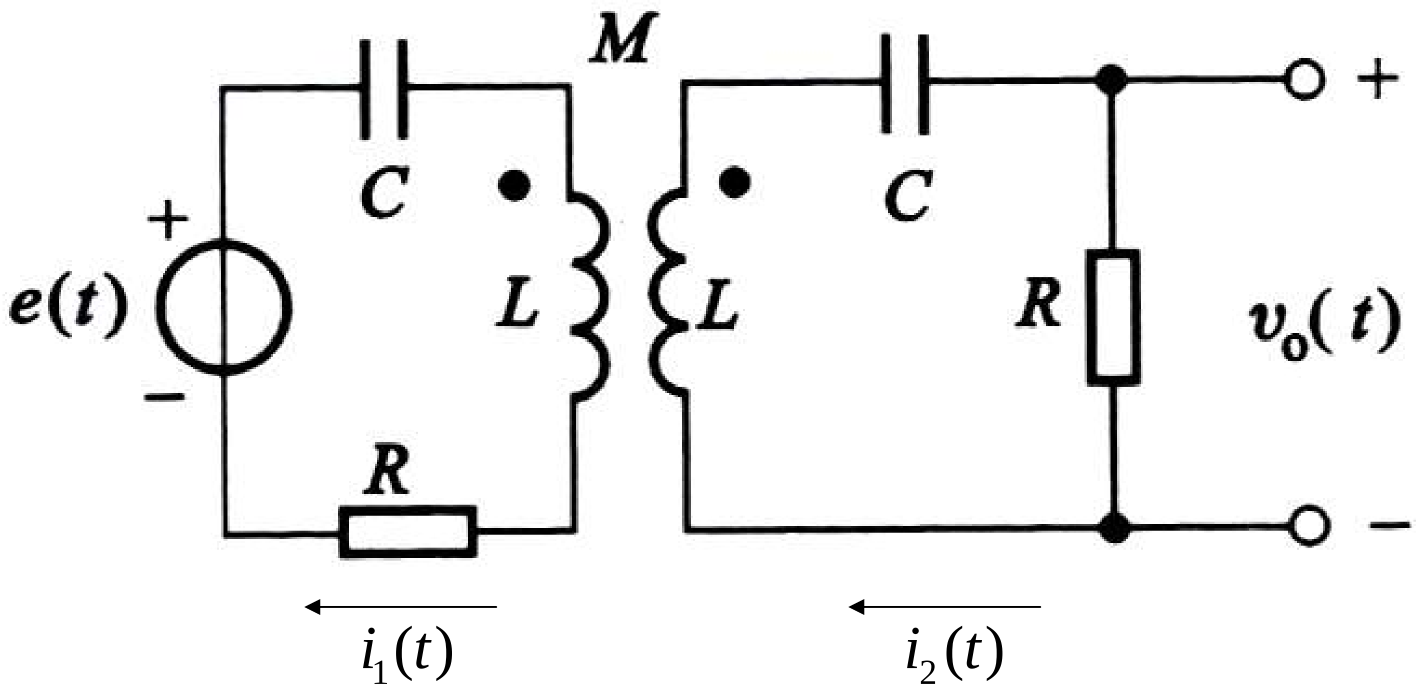
KCL

$$i_c = c\frac{dV_c(t)}{dt}$$

$$i_G = GV_R$$

$$i_L = \frac{1}{L}\int_{-\infty}^{t} V_L(\tau) d\tau$$

- 例:列出所示电路的常系数微分方程式:



$$\begin{cases} \frac{1}{c}\int_{-\infty}^{t} i_1 dt + L\frac{di_1}{dt} + Ri_1 - M\frac{di_2}{dt} = e(t) & (1) \\ \frac{1}{c}\int_{-\infty}^{t} i_2 dt + L\frac{di_2}{dt} + Ri_2 - M\frac{di_1}{dt} = 0 & (2) \\ v_o = Ri_2 & (3) \end{cases}$$

$$\frac{d^3}{dt^3}(1), \frac{d}{dt}(2) \longrightarrow \begin{cases} \frac{1}{c} i_1^{(2)} + Li_1^{(4)} + Ri_1^{(3)} - Mi_2^{(4)} = e^{(3)} \\ \frac{1}{c} i_2 + Li_2^{(2)} + Ri_2^{(1)} - Mi_1^{(2)} = 0 \end{cases}$$

$$\begin{cases} i_1^{(2)} = \frac{1}{CM} i_2 + \frac{L}{M} i_2^{(2)} + \frac{R}{M} i_2^{(1)} \\ i_1^{(3)} = \frac{1}{CM} i_{2^{(1)}} + \frac{L}{M} i_2^{(3)} + \frac{R}{M} i_2^{(2)} \\ i_1^{(4)} = \frac{1}{CM} i_2^{(2)} + \frac{L}{M} i_2^{(4)} + \frac{R}{M} i_2^{(2)} \end{cases}$$

$$(L^2 - M^2)\frac{d^4 i_2}{dt^4} + 2RL\frac{d^3 i_2}{dt^3} + (R^2 + \frac{2L}{C})\frac{d^2 i_2}{dt^2}$$

$$+\frac{2R}{C}\frac{di_2}{dt} + \frac{1}{C^2}i_2 = M\frac{d^3 e(t)}{dt^3}$$

如果该电路无电容，$C \to \infty$

$$(L^2 - M^2)\frac{d^4 i_2}{dt^4} + 2RL\frac{d^3 i_2}{dt^3} + R^2\frac{d^2 i_2}{dt^2} = M\frac{d^3 e(t)}{dt^3}$$

$$(L^2 - M^2)\frac{d^2 i_2}{dt^4} + 2RL\frac{di_2}{dt} + R^2 i_2 = M\frac{de(t)}{dt}$$

导纳型电网络，采用KCL，$\sum i_k = 0$

$$i_c = c\frac{dv_c}{dt}$$

$$i_G = GV_R$$

$$i_L = \frac{1}{L}\int_{-\infty}^{t} v_L(\tau)d\tau$$

- 线性时不变系统，输入e(t),响应r(t)

$$C_0\frac{d^n r(t)}{dt^n}+C_1\frac{d^{n-1}r(t)}{dt^{n-1}}+\ldots+C_{n-1}\frac{dr(t)}{dt}+C_n r(t)$$

$$=E_0\frac{d^m e(t)}{dt^m}+E_1\frac{d^{m-1}e(t)}{dt^{m-1}}+\ldots+E_{m-1}\frac{de(t)}{dt}+E_m e(t)$$

- 完全解=通解+特解

↓

$$r_g(t)=Ae^{\alpha t}$$

↓

$$C_0\alpha^n+C_1\alpha^{n-1}+\ldots+C_{n-1}\alpha+C_n=0$$

↓ ↓ K重根

$$r_g(t)=A_1e^{\alpha_1 t}+A_2e^{\alpha_2 t}+\ldots+A_ne^{\alpha_n t}$$

$$r_g(t)=(A_0+A_1t+\ldots+A_{k-1}t^{k-1})e^{\alpha t}$$

- 例：求微分方程的通解

$$\frac{d^3r(t)}{dt^3}+8\frac{d^2r(t)}{dt^2}+21\frac{dr(t)}{dt}+18r(t)=e(t)$$

$$\alpha^3+8\alpha^2+21\alpha+18=0$$

$$(\alpha+2)(\alpha+3)^2=0$$

$$\alpha_1=-2,\alpha_{2,3}=-3(\text{二重根})$$

$$\mathrm{r_g}(t)=A_1e^{-2t}+A_2e^{-3t}+A_3te^{-3t}$$

- 特解的形式

激励函数$e(t)$	响应函数$r(t)$的特解
E（常数）	B
t^p	$B_1 t^p + B_2 t^{p-1} + \cdots + B_p t + B_{p+1}$
e^{at}	Be^{at}
$\cos(\omega t)$	$B_1\cos(\omega t) + B_2\sin(\omega t)$
$\sin(\omega t)$	
$t^p e^{at}\cos(\omega t)$	$(B_1 t^p + \cdots + B_p t + B_{p+1})e^{at}\cos(\omega t)$
$t^p e^{at}\sin(\omega t)$	$+ (D_1 t^p + \cdots + D_p t + D_{p+1})e^{at}\sin(\omega t)$

- 例：求解微分方程：

$$\frac{d^2 r(t)}{dt^2} - 2\frac{dr(t)}{dt} - 3r(t) = 3t + 1$$

(1)通解

$$\alpha^2 - 2\alpha - 3 = 0$$

$$(\alpha + 1)(\alpha - 3) = 0$$

$$r_g(t) = A_1 e^{-t} + A_2 e^{3t}$$

(2)特解

$$B(t) = B_1 t + B_2$$

$$-2B_1 - 3B_1 t - 3B_2 = 3t + 1$$

$$-3B_1 = 3 \rightarrow B_1 = -1$$

$$-2B_1 - 3B_2 = 1 \rightarrow B_2 = 1/3$$

$$\therefore r(t) = r_g(t) + B(t) = A_1 e^{-t} + A_2 e^{3t} - t + 1/3$$

- 求待定系数A_1，A_2，…，A_n

$$r(t) = A_1 e^{\alpha_1 t} + A_2 e^{\alpha_2 t} + \ldots + A_n e^{\alpha_n t} + B(t)$$

$$\begin{bmatrix} 1 & 1 & \ldots & 1 \\ \alpha_1 & \alpha_2 & \ldots & \alpha_n \\ \vdots & \vdots & \vdots & \vdots \\ \alpha_1^{n-1} & \alpha_2^{n-1} & \ldots & \alpha_n^{n-1} \end{bmatrix} \times \begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ A_n \end{bmatrix} = \begin{bmatrix} r(0) - B(0) \\ r^{(1)}(0) - B^{(1)}(0) \\ \vdots \\ r^{(n-1)}(0) - B^{(n-1)}(0) \end{bmatrix}$$

↑

范德蒙特矩阵V

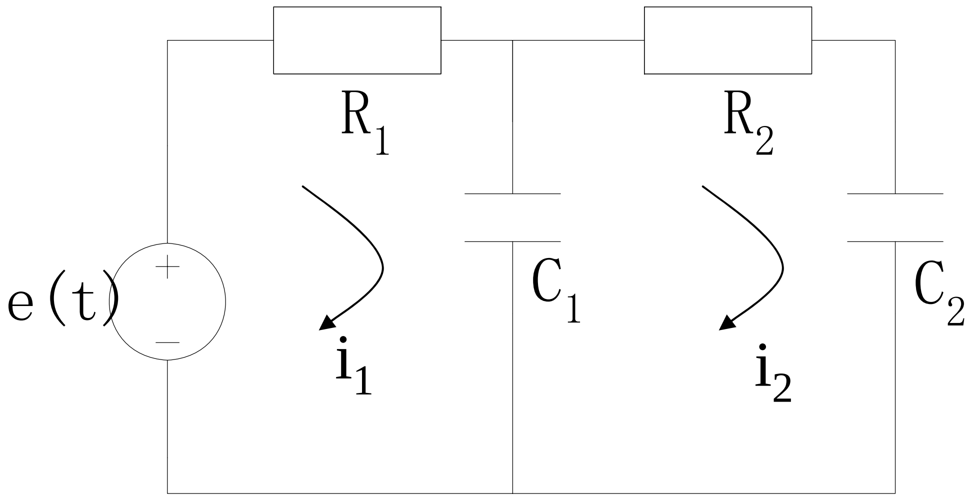
$$V \times A = [r^{(k)}(0) - B^{(k)}(0)]$$

$$A = V^{-1}[r^{(k)}(0) - B^{(k)}(0)]$$

- 例：已知 $e(t)=\sin 2tU(t) \quad R_1=R_2=1\Omega, \quad C_1=1/2F$

$$C_2=\frac{1}{3}F, \quad V_1(t)|_{t=0}=0 \quad V_2(t)|_{t=0}=0$$

求：V_2(t)的表达式



- 建立微分方程：

$$e(t) = R_1 i_{总} + R_2 i_{c2} + v_2(t)$$

$$R_1 R_2 C_1 C_2 V_2''(t) + (R_1 C_1 + R_1 C_2 + R_2 C_2) V_2'(t) + V_2(t) = e(t)$$

$$\frac{1}{6} V_2''(t) + \frac{7}{6} V_2'(t) + V_2(t) = \sin 2t U(t)$$

- 求通解 $V_g(t) = A_1 e^{-t} + A_2 e^{-6t}$
- 求特解 $B(t) = B_1 \sin 2t + B_2 \cos 2t$
- 完全解 $V(t) = A_1 e^{-t} + A_2 e^{-6t} + \frac{3}{50} \sin 2t - \frac{21}{50} \cos 2t$
- 代入初始条件

$$V(t) = \frac{12}{25} e^{-t} - \frac{3}{50} e^{-6t} + \frac{3}{50} \sin 2t - \frac{21}{50} \cos 2t$$

2.3　起始点的跳变

- 初始条件—A_1，A_2，…, A_n
  - 起始条件发生跳变→初始条件要重新确定
  - 零输入及零状态响应、LT可不考虑跳变
- 起始状态和初始状态
  - 起始状态(0-)：施加激励前一瞬间起始时刻
  - 初始状态(0+)：施加激励后的初始时刻
- 起始条件与初始条件
  - 起始条件：　$r^{(k)}(0-)$
  - 初始条件：　$r^{(k)}(0+)$

$$r^{(k)}(0-) = r^{(k)}(0+)$$

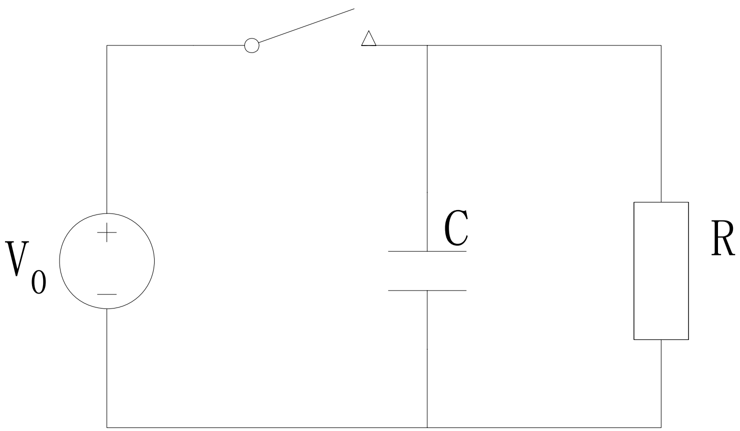
$r^{(k)}(t)$在$t = 0$处连续

$$r^{(k)}(0-) \neq r^{(k)}(0+)$$

$r^{(k)}(t)$在$t = 0$处跳变

初始条件为：$r^{(k)}(0+)$

- 初始条件的确定
  - 贮能器件有无跳变(电容、电感)
    - 无冲激电流(阶跃电压)作用电容，$v_c(t)$连续
    - 无冲激电压(阶跃电流)作用电感，$i_L(t)$连续



$$\mathrm{RC}\frac{\mathrm{dVc}}{\mathrm{dt}}+V_c=0$$

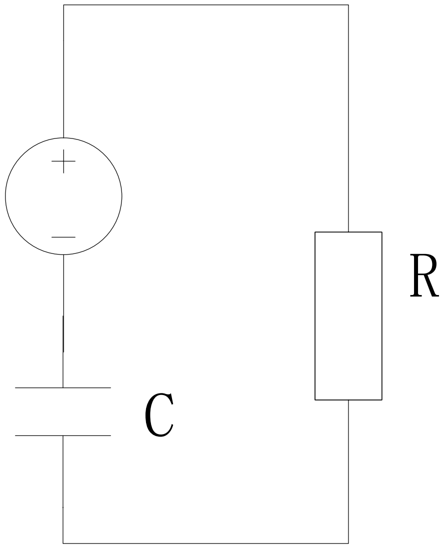
$$RC\alpha+1=0,\alpha=-1/RC,$$

$$V_c(t)=Ae^{-\frac{1}{RC}t}$$

$$V_c(0^-)=V_0$$

$$V_c(0^+)=V_0$$

$$V_c(t)=V_0e^{-\frac{t}{RC}}$$



$$RC\frac{dV_c(t)}{dt}+V_c(t)=\delta(t)$$

$$V_c(0-)=0$$

通解： $V_c(t)=Ae^{-\frac{1}{RC}t}$

$t=0+,\delta(t)=0,$特解为0

$$\int_{0-}^{0+}(RC\frac{dVc}{dt}+Vc)dt=\int_{0-}^{0+}\delta(t)dt=1$$

$$RC(V_c(0+)-V_c(0-))+\int_{0-}^{0+}V_c dt=1$$

$$\int_{0-}^{0+}V_c dt=0,V_c(0^+)-V_c(0-)=\frac{1}{RC}$$

$$V_c(0^+)=\frac{1}{RC},V_c(t)=\frac{1}{RC}e^{-\frac{t}{RC}}u(t)$$

δ函数匹配法

$V_c(t)$不可能有$\delta(t)$,因为右边仅有$\delta(t)$

$$RC\frac{dV_c}{dt}=\delta(t)$$

$$V_c(0^+)=\frac{1}{RC},V_c(t)=\frac{1}{RC}e^{-\frac{t}{RC}}u(t)$$

$$i_c(t) = C\frac{dV_c(t)}{dt}$$

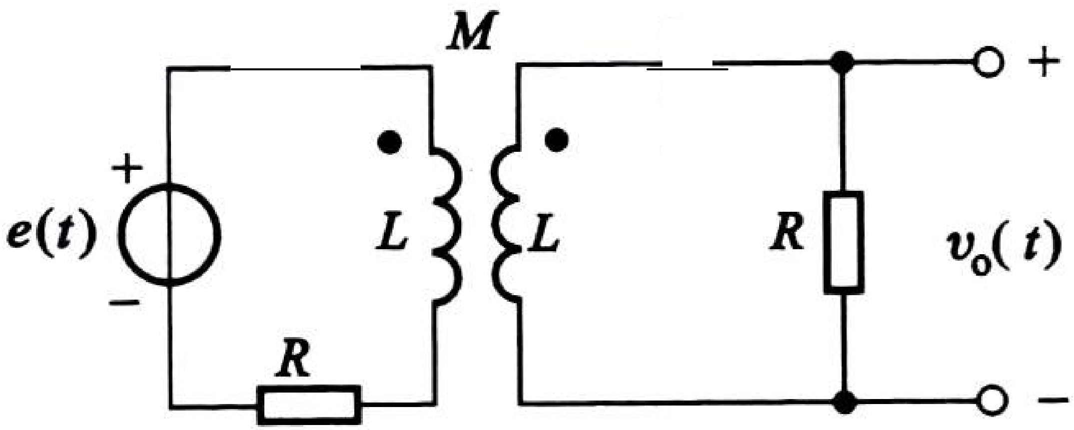
$$= C\frac{d}{dt}[\frac{1}{RC}e^{-\frac{1}{RC}t}u(t)]$$

$$= \frac{1}{R}\delta(t) - \frac{1}{R^2C}e^{-\frac{1}{RC}t}u(t)$$

例：微分方程： $(L^2 - M^2)\frac{d^2 i_2}{dt^2} + 2RL\frac{di_2}{dt} + R^2 i_2 = M\frac{de}{dt}$

若e(t)加入单位阶跃电压，,系统无储能，

$i_2(0^-) = 0, i_2{}'(0^-) = 0$　　求i_2(t)



$$(L^2 - M^2)\alpha^2 + 2RL\alpha + R^2 = 0$$

$$\alpha_{1,2} = \frac{-R}{L \pm M}$$

$$i_{2g}(t) = A_1 e^{-\frac{R}{L+M}t} + A_2 e^{-\frac{R}{L-M}t}$$

$$M\frac{du(t)}{dt} = M\delta(t)$$

$$M\frac{du(t)}{dt} = M\delta(t)$$

$$B(t) = 0$$

$$i_2(t) = A_1 e^{-\frac{R}{L+M}t} + A_2 e^{-\frac{R}{L-M}t}$$

$$(L^2 - M^2)\frac{d^2 i_2}{dt^2} + 2RL\frac{di_2}{dt} + R^2 i_2 = M\delta(t)$$

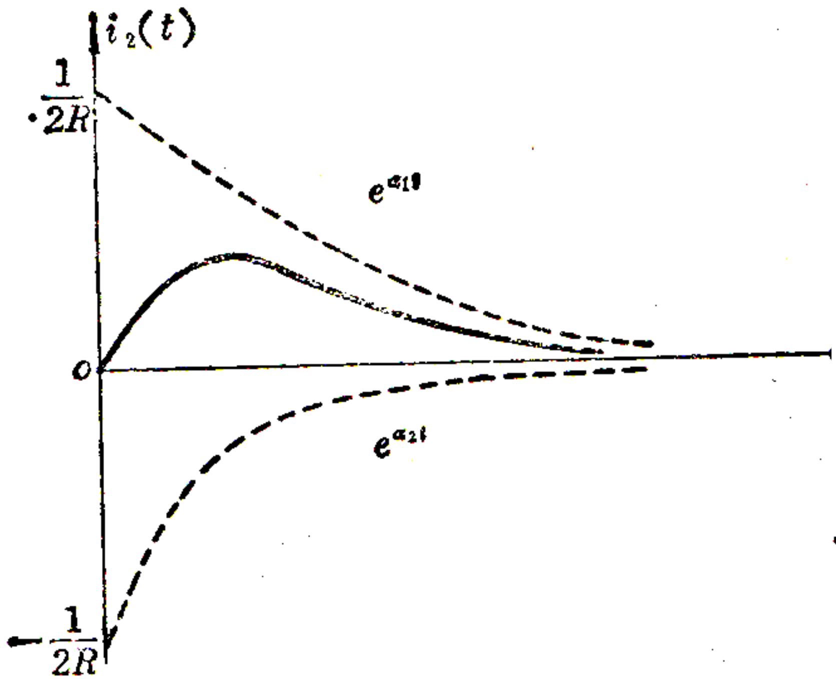
$$(L^2 - M^2) i_2^{(2)} = M\delta(t)$$

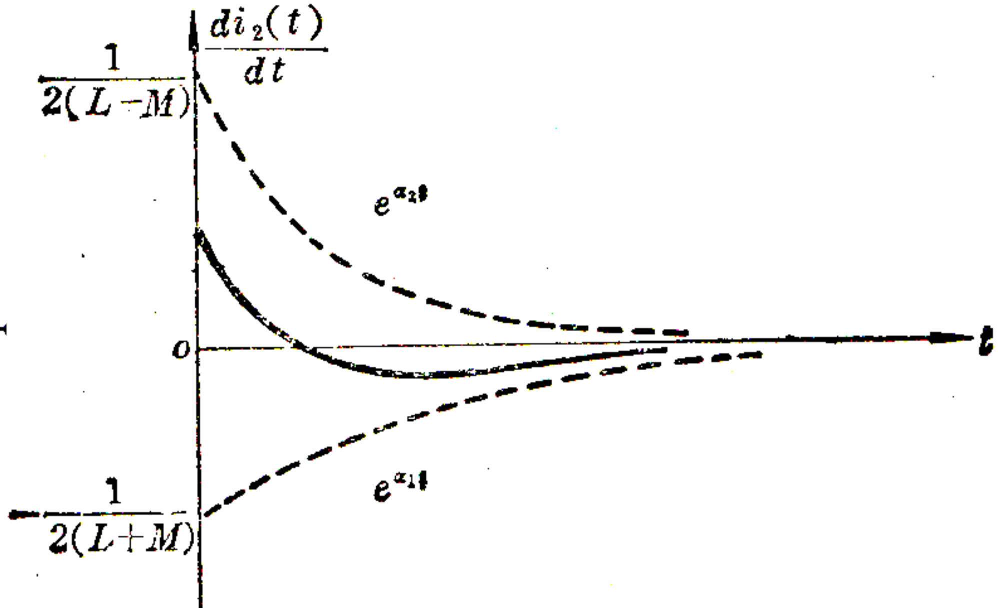
$$\begin{cases} i_2(0+) = 0 \\ i_2'(0+) = \dfrac{M}{L^2 - M^2} \end{cases}$$

$$\longrightarrow \begin{cases} A_1 + A_2 = 0 \\ \alpha_1 A_1 + \alpha_2 A_2 = \dfrac{M}{L^2 - M^2} \end{cases} \longrightarrow \begin{cases} A_1 = 1/2R \\ A_2 = -1/2R \end{cases}$$

$$i_2(t) = \frac{1}{2R}\left(e^{-\frac{R}{L+M}t} - e^{-\frac{R}{L-M}t}\right)$$

$$i_2{}'(t) = \frac{1}{2}\left(-\frac{1}{L+M}e^{-\frac{R}{L+M}t} + \frac{1}{L-M}e^{-\frac{R}{L-M}t}\right)U(t)$$





- 例： 微分方程为：$r'''(t)+2r''(t)+3r'(t)+4r(t)=\delta''(t)+5\delta(t)$

试求跳变量 $r_{zs}(0+), r'_{zs}(0+), r''_{zs}(0+)$

$$r'''(t)+2r''(t)+3r'(t)+4r(t)=\delta''(t)+5\delta(t)$$

$\delta''(t)$ $\delta'(t)$ $\delta(t)$ $\varepsilon(t)$

$2\delta'(t)$ $3\delta(t)$

$-2\delta'(t)$ $-2\delta(t)$ $-2\varepsilon(t)$

$-4\delta(t)$

$6\delta(t)$ → $6\varepsilon(t)$

$$\begin{cases} r_{zs}(0+)=1 \\ r'_{zs}(0+)=-2 \\ r''_{zs}(0+)=6 \end{cases}$$

2.4 零输入响应与零状态响应

LTI系统

- 系统响应
  - 自由响应
  - 强迫响应

$$r(t)=\sum_{i=1}^{n}A_i e^{\alpha_i t}+B(t)$$

- 系统响应
  - 零输入响应
  - 零状态响应

$$r(t)=r_{zi}(t)+r_{zs}(t)$$

$r_{zi}(t)$:外加激励为0时，由起始状态引起的响应

$r_{zs}(t)$:起始状态为0时，由激励引起的响应

$$r_{ZI}(t)=\sum_{j=1}^{n}A_{ZIj}e^{\alpha_j t}$$

$$e(t)=0\rightarrow r_{ZI}^{(k)}(0+)=r_{ZI}^{(k)}(0-)\rightarrow A_{ZIj}$$

A_{ZIj}由$r_{ZI}^{(k)}(0-)$决定，由起始条件决定

$$r_{ZS}(t)=\sum_{i=1}^{n}A_{ZSi}e^{\alpha_i t}+B(t)$$

$$r_{ZS}^{(k)}(0+)=r^{(k)}(0+)-r^{(k)}(0-)\rightarrow A_{ZSj}$$

A_{ZSj}由$r_{ZS}^{(k)}(0+)$和$B^{(k)}(0+)$确定

$$\mathrm{r}_{\mathrm{zs}}(t)=\sum_{i=1}^{n}A_{zsj}e^{\alpha_j t}+B(t)\Big|_{t=0+}=r_{zs}^{(k)}(0+)$$

$$A_{zs}=V^{-1}[r_{zs}^{(k)}(0+)-B^{(k)}(0+)]$$

$$=V^{-1}[r^{(k)}(0+)-r^{(k)}(0-)-B^{(k)}(0+)]$$

$$=V^{-1}[-B^{(k)}(0+)]$$（如果起始状态无跳变）

$$V=\begin{bmatrix}1 & \dots & \dots & 1\\ \alpha_1 & \alpha_2 & \dots & \alpha_n\\ \dots & \dots & \dots & \dots\\ \alpha_1^{n-1} & \alpha_2^{n-1} & \dots & \alpha_n^{n-1}\end{bmatrix}$$

$$r(t) = r_{zI}(t) + r_{zs}(t)$$

$$= \sum_{j=1}^{n} A_j e^{\alpha_j t} + B(t)$$

自由响应　强迫响应

$$= \sum_{j=1}^{n} A_{ZIj} e^{\alpha_j t} + \sum_{j=1}^{n} A_{Zsj} e^{\alpha_j t} + B(t)$$

零输入响应　　　零状态响应

- 例：已知系统微分方程为：

$$2\frac{d^2r(t)}{dt^2}+3\frac{dr(t)}{dt}+r(t)=10e^{-3t}$$

系统无跳变，起始条件=初始条件，$r(0-)=1, r'(0-)=0$

求 $r_{ZI}(t), r_{ZS}(t), r(t)$

如果激励信号加倍，响应是否加倍？

- 零输入响应

$$2\alpha^2 + 3\alpha + 1 = 0$$

$$\alpha_1 = -1/2, \alpha_2 = -1$$

$$r_{zi}(t) = A_{zi1}e^{-t} + A_{zi2}e^{-\frac{1}{2}t}$$

$$r_{zi}(0+) = 1, r'_{zi}(0+) = 0$$

$$A_{zi1} = -1, A_{zi2} = 2$$

$$r_{zi}(t) = -e^{-t} + 2e^{-\frac{1}{2}t}$$

- 求特解

$$Be^{-3t}$$

$$18B-9B+B=10$$

$$B=1$$

- 求零状态响应

$$r_{zs}(t)=A_{zs1}e^{-t}+A_{zs2}e^{-t/2}+e^{-3t}$$

$$r^{(k)}(0+)=0$$

$$A_{zs1}=-5, A_{zs2}=4$$

$$r_{zs}(t)=-5e^{-t}+4e^{-t/2}+e^{-3t}$$

$$r(t) = r_{zi}(t) + r_{zs}(t)$$

$$r(t) = \underbrace{-e^{-t} + 2e^{-t/2}}_{\text{零输入}} \underbrace{-5e^{-t} + 4e^{-t/2} + e^{-3t}}_{\text{零状态}}$$

$$r(t) = -6e^{-t} + 6e^{-t/2} + e^{-3t}$$

- 若激励信号加倍
  - 零输入响应不变

$$r_{zi}(t) = -e^{-t} + 2e^{-\frac{1}{2}t}$$

  - 零状态响应

$$Be^{-3t}$$

$$18B - 9B + B = 20$$

$$B = 2$$

$$r_{zs}(t) = A_{zs1}e^{-t} + A_{zs2}e^{-t/2} + 2e^{-3t}$$

$$r^{(k)}(0+) = 0$$

$$A_{zs1} = -10, A_{zs2} = 8$$

$$r_{zs}(t) = -10e^{-t} + 8e^{-t/2} + 2e^{-3t}$$

  - 完全响应 $r(t) = r_{zi}(t) + r_{zs}(t)$

$$r(t) = -e^{-t} + 2e^{-t/2} - 10e^{-t} + 8e^{-t/2} + 2e^{-3t}$$

$$= -11e^{-t} + 10e^{-t/2} + 2e^{-3t}$$

- 若初始条件加倍
  - 零输入响应加倍

$$r_{zi}(t) = -2e^{-t} + 4e^{-\frac{1}{2}t}$$

  - 零状态响应不变

$$r_{zs}(t) = -5e^{-t} + 4e^{-t/2} + e^{-3t}$$

  - 完全响应

$$r(t) = r_{zi}(t) + r_{zs}(t)$$

$$r(t) = -2e^{-t} + 4e^{-t/2} - 5e^{-t} + 4e^{-t/2} + 2e^{-3t}$$

$$= -7e^{-t} + 8e^{-t/2} + e^{-3t}$$

常系数微分方程的解＝零输入响应＋零状态响应
零输入响应和零状态响应分别呈线性

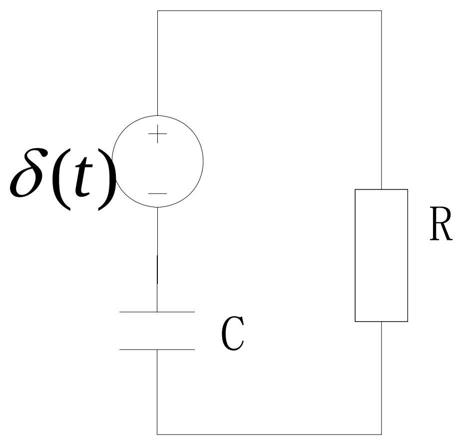
零输入响应的求解简单：齐次方程－＞代数方程
－＞起始条件定待定系数

零状态响应的求解较烦：先求特解，再定待定系数
当起始状态有跳变，还必须确定初始条件

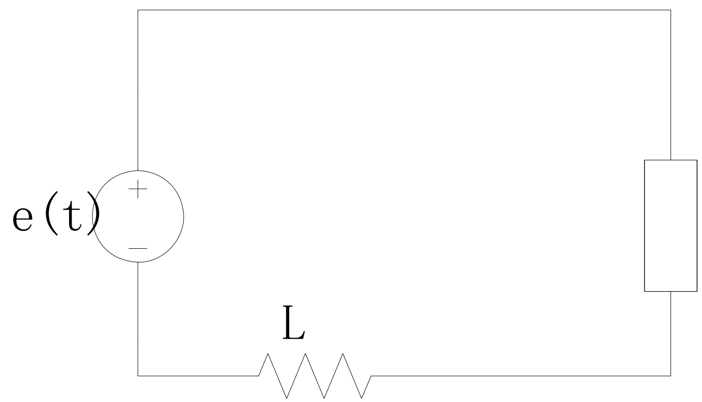
后面将介绍卷积法求零状态响应

2.5　冲激响应与阶跃响应

- 定义：系统在单位冲激信号作用下的零状态响应称为单位冲激响应
- 冲激响应反映系统固定性质

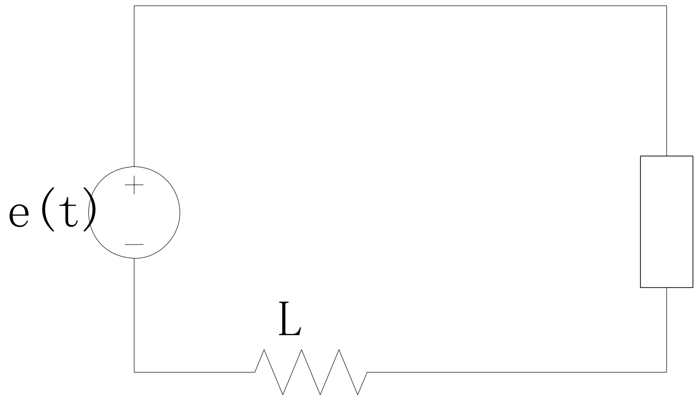


$$V_c(t)=\frac{1}{RC}e^{-\frac{t}{RC}}u(t)$$



$$i_L(t)=\frac{1}{L}e^{-\frac{Rt}{L}}u(t)$$

- RL电路，零状态，在单位冲激电压下流过电感的电流



$$Ri_L(t)+L\frac{di}{dt}=\delta(t),i_L(0-)=0$$

$$[i_L(0+)-i_L(0-)]=1/L,i_L(0+)=1/L$$

$$L\alpha+R=0,\alpha=-R/L$$

$$i_L(t)=Ae^{-\frac{R}{L}t},A=1/L$$

$$i_L(t)=\frac{1}{L}e^{-\frac{R}{L}t}u(t)$$

$$V_L(t)=L\frac{di}{dt}=\delta(t)-\frac{R}{L}e^{-\frac{Rt}{L}}u(t)$$

- 由于冲激电源在t=0-瞬间具有无限强烈的瞬时值
- 所以在极短时间把能量传输给贮能器件即$v_c(0-),i_L(0-)$跃变为$v_c(0+),i_L(0+)$
- 然后冲激电源的作用消失，$V_c(t),i_L(t)$指数衰减
- 在t=0瞬间以后冲激响应实际上是只有初始贮能$v_c(0+)$或$i_L(0+)$的零输入响应

$$h(t) = L[\delta(t)]$$

$$h(t) = Ae^{\alpha t}u(t)$$（一阶电路）

$$\mathrm{h(t)} = \sum_{\mathrm{i=1}}^{\mathrm{n}} \mathrm{A_i} e^{\alpha_i t}u(t)$$（n阶电路）

- LTI系统的冲激响应

$$C_0\frac{d^n r(t)}{dt^n}+C_1\frac{d^{n-1}r(t)}{dt^{n-1}}+\ldots+C_{n-1}\frac{dr(t)}{dt}+C_n r(t)$$

$$=E_0\frac{d^m e(t)}{dt^m}+E_1\frac{d^{m-1}e(t)}{dt^{m-1}}+\ldots+E_{m-1}\frac{de(t)}{dt}+E_m e(t)$$

$e(t)=\delta(t)$代入方程

一般$n>m$，左边的$\frac{d^n r}{dt^n}$应包含$\delta^{(m)}(t)$

$n=m+1, r'(t)$对应$\delta(t), r(t)$不包含$\delta(t)$及其导数项

$$n>m,(n=m+1, r'(t)\rightarrow\delta(t))$$

$$n=m, r(t)\rightarrow\delta(t)$$

$$n<m,(n=m-1, r(t)\rightarrow\delta'(t))$$

$n > m$时

$\delta(t)$及其导数在$t > 0$时为0，方程式右为0

冲激响应与齐次方程解相同

$$h(t)=\sum_{i=1}^{n} A_i e^{\alpha_i t} u(t)$$

待定常数由方程式两端奇异函数匹配法求得

n=m时，将包含一项δ(t)

n<m时，将包含δ(t)导数项

例：某系数的微分方程为

$$r^{(2)}(t)+5r^{(1)}(t)+6r(t)=e^{(1)}(t)+4e(t)$$

求$h(t)$

$$\alpha^2 + 5\alpha + 6 = 0$$

$$(\alpha + 2)(\alpha + 3) = 0$$

$$h(t) = (A_1 e^{-2t} + A_2 e^{-3t})u(t)$$

$$h'(t) = (A_1 + A_2)\delta(t) + (-2A_1 e^{-2t} - 3A_2 e^{-3t})u(t)$$

$$h''(t) = (A_1 + A_2)\delta'(t) + (-2A_1 - 3A_2)\delta(t) + (4A_1 e^{-2t} + 9A_2 e^{-3t})u(t)$$

等式左边为

$$(A_1 + A_2)\delta'(t) + (3A_1 + 2A_2)\delta(t) + 0u(t)$$

$$A_1 = 2, A_2 = -1$$

$$h(t) = (2e^{-2t} - e^{-3t})u(t)$$

- 阶跃响应：系统在单位阶跃信号作用下的零状态响应
- 冲激响应与阶跃响应可相互求得

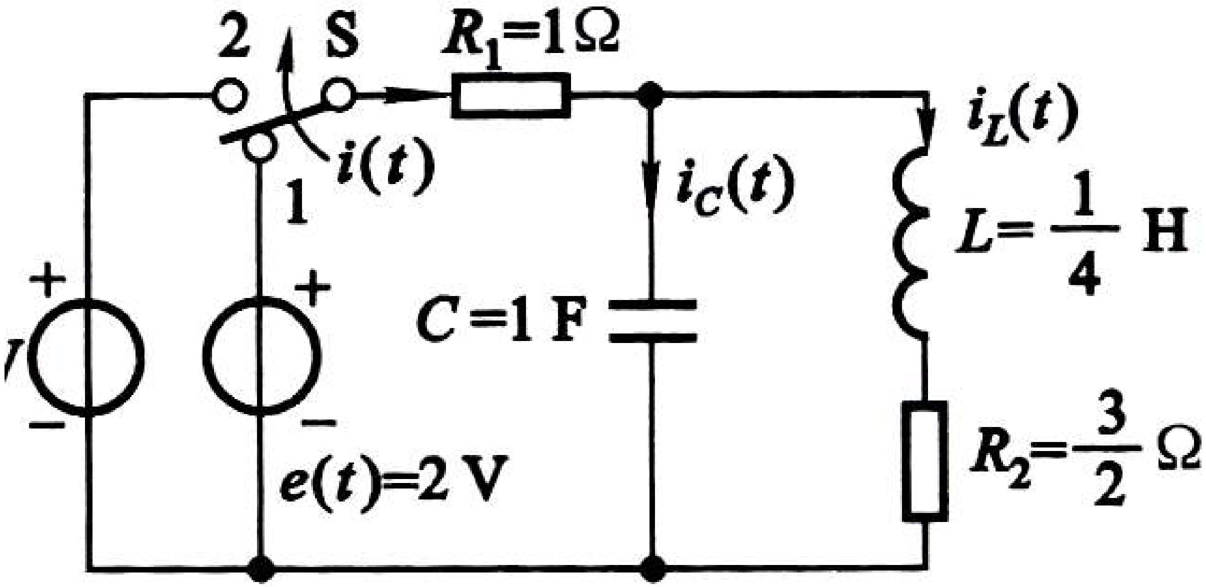
$$g(t) = L[u(t)]$$

$$\delta(t) = \frac{du(t)}{dt}$$

$$h(t) = L[\delta(t)] = L[\frac{du(t)}{dt}] = \frac{d}{dt}[L[u(t)]] = \frac{d}{dt}g(t)$$

$$g(t) = \int_{-\infty}^{t} h(\tau)d\tau$$

- 例：给定如图所示电路，t<0开关S处于“1”的位置，而且已经达到稳态；当t=0时，S由“1”转向“2”。求电流i(t)的冲激响应。



- 微分方程： $\frac{d^2 i}{dt^2} + 7\frac{di}{dt} + 10i = \frac{d^2 e}{dt^2} + 6\frac{de}{dt} + 4e$
- 冲激响应：

$h''(t) + 7h'(t) + 10h(t) = \delta''(t) + 6\delta'(t) + 4\delta(t)$ $\quad h(t) = (A_1 e^{-2t} + A_2 e^{-5t})U(t)$

$\delta''(t)$ $\quad \delta'(t)$ $\quad \delta(t)$

$7\delta'(t)$ $\quad 10\delta(t)$ $\longrightarrow \begin{cases} h(0+) = -1 \\ h'(0+) = 1 \end{cases} \longrightarrow \begin{cases} A_1 = -4/3 \\ A_2 = 1/3 \end{cases}$

$-\delta'(t)$ $\quad -\delta(t)$ $\quad -\varepsilon(t)$

$-7\delta(t)$

$\delta(t)$ $\quad \varepsilon(t)$

$$h(t) = (-\frac{4}{3}e^{-2t} + \frac{1}{3}e^{-5t})U(t) + \delta(t)$$

最高阶项平衡－＞对低阶项影响－＞低阶项平衡

- 微分方程： $\frac{d^2 i}{dt^2}+7\frac{di}{dt}+10i=\frac{d^2 e}{dt^2}+6\frac{de}{dt}+4e$
- 阶跃响应：

$$g''(t)+7g'(t)+10g(t)=\delta'(t)+6\delta(t)+4U(t) \qquad g(t)=A_1e^{-2t}+A_2e^{-5t}+B(t)$$

$\delta'(t)$ $\quad\delta(t)$ $\quad\varepsilon(t)$

$\downarrow$

$7\delta(t)$

$-\delta(t)$ $\quad-\varepsilon(t)$

$$\longrightarrow \begin{cases} g(0+)=1 \\ g'(0+)=-1 \end{cases} \longrightarrow \begin{cases} A_1=2/3 \\ A_2=-1/15 \end{cases}$$

$$g(t)=(\frac{2}{3}e^{-2t}-\frac{1}{15}e^{-5t}+\frac{2}{5})U(t)$$

2.6 卷积

- 用δ(t)表示任意信号
  - 任一信号可以由一系列矩形窄脉冲表示

$$f(t)=\sum_{k=-\infty}^{\infty} f(k\Delta\tau)[u(t-k\Delta\tau)-u(t-k\Delta\tau-\Delta\tau)]$$

$$\Delta\tau\rightarrow 0, u(t)-u(t-\Delta\tau)=\frac{du(t)}{dt}\Delta\tau=\delta(t)\Delta\tau$$

$$f(t)=\lim_{\Delta\tau\rightarrow 0}\sum_{k=-\infty}^{\infty} f(k\Delta\tau)\delta(t-k\Delta\tau)\Delta\tau$$

$$f(t)=\int_{-\infty}^{\infty} f(\tau)\delta(t-\tau)d\tau$$

2.6　卷积

- 求系统零状态响应
  - 输入信号分解为许多冲激信号

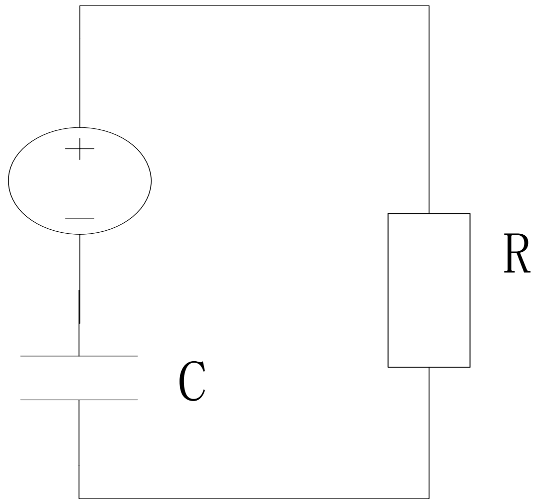
$$e(t)=\int_{-\infty}^{\infty}e(\tau)\delta(t-\tau)d\tau$$

  - 线性系统的比例性均匀性

$$r(t)=\int_0^t e(\tau)h(t-\tau)d\tau$$

$$=e(t)*h(t)$$

例：如图一个RC电路，激励电压为 $e(t)=V_0(1-e^{-\beta t})u(t)$
求输入电压$V_R(t)$的零状态响应



电流的冲激响应：

$$i_c=\frac{1}{R}\delta(t)-\frac{1}{R^2c}e^{-\frac{1}{Rc}t}u(t)$$

R上冲激响应为：

$$h(t)=i_cR=\delta(t)-\frac{1}{Rc}e^{-\frac{1}{Rc}t}u(t)$$

$$r_{zs}=\int_0^t e(\tau)h(t-\tau)d\tau$$

$$=\int_0^t V_0(1-e^{-\beta\tau})[\delta(t-\tau)-\frac{1}{Rc}e^{-\frac{1}{Rc}(t-\tau)}u(t-\tau)]d\tau$$

$$=\int_0^t V_0(1-e^{-\beta\tau})\delta(t-\tau)d\tau-\int_0^t -\frac{V_0}{Rc}(1-e^{-\beta\tau})e^{-\frac{1}{Rc}(t-\tau)}d\tau$$

$$=V_0(1-e^{-\beta t})-[V_0(1-e^{-\frac{1}{Rc}t})-\frac{V_0}{1-Rc\beta}(e^{-\beta t}-e^{-\frac{t}{Rc}})$$

- 系统完全响应零状态响应

$$r(t) = r_{zi}(t) + r_{zs}(t)$$

$$= \sum_{j=1}^{n} A_{zij} e^{\alpha_j t} + \int_{0-}^{t} e(\tau)\delta(t-\tau)d\tau$$

– 卷积积分上下限说明

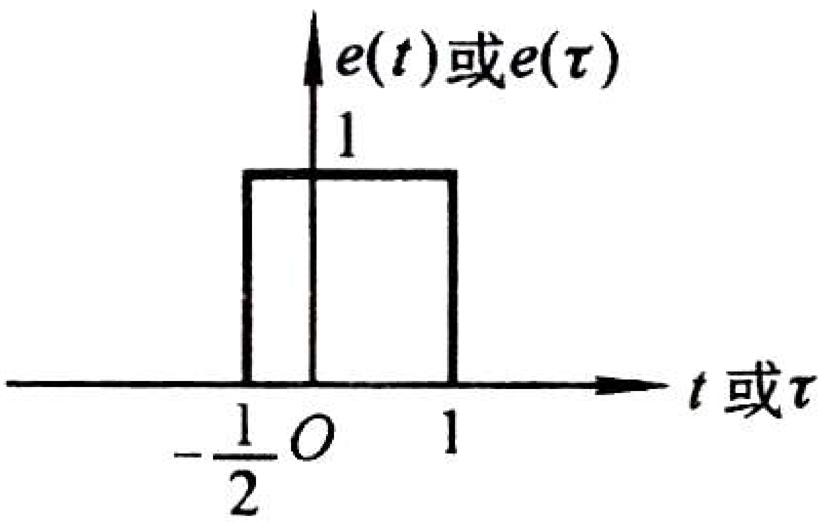
$$s(t) = \int_{-\infty}^{\infty} f_1(\tau) f_2(t-\tau)d\tau$$

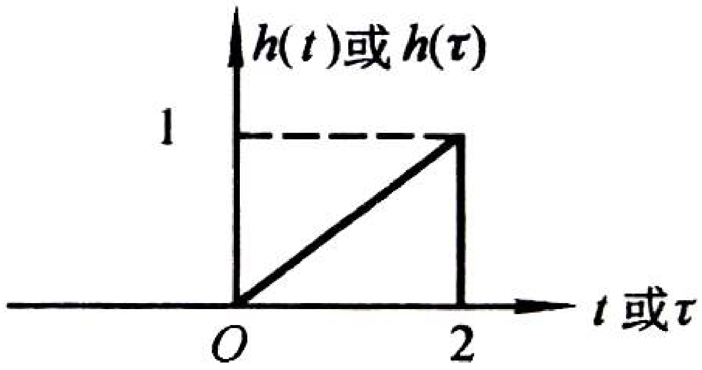
如$t < 0,\ f_1(t) = 0,$则$\int_{-\infty}^{0} f_1(\tau) f_2(t-\tau)d\tau = 0$

如$t < 0,\ f_2(t) = 0, t-\tau < 0$时$f_2(t-\tau) = 0$则$\int_{t}^{\infty} f_1(\tau) f_2(t-\tau)d\tau = 0$

因果信号$\int_{0}^{t}$

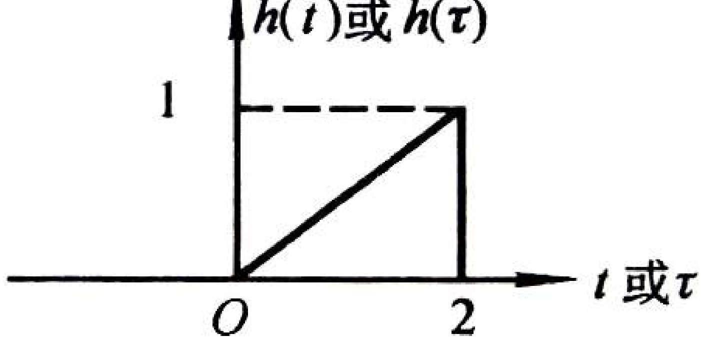
- 卷积 e(t)*h(t)

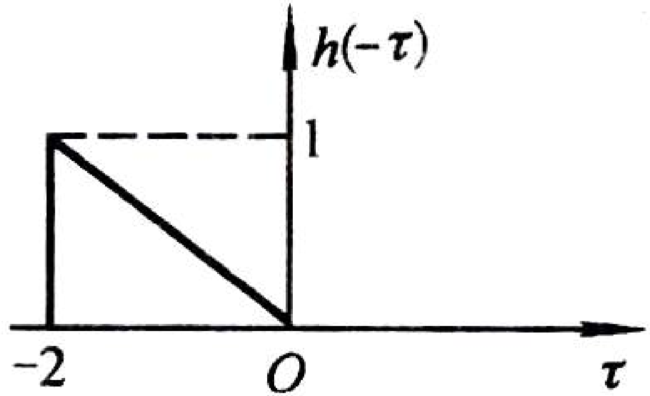


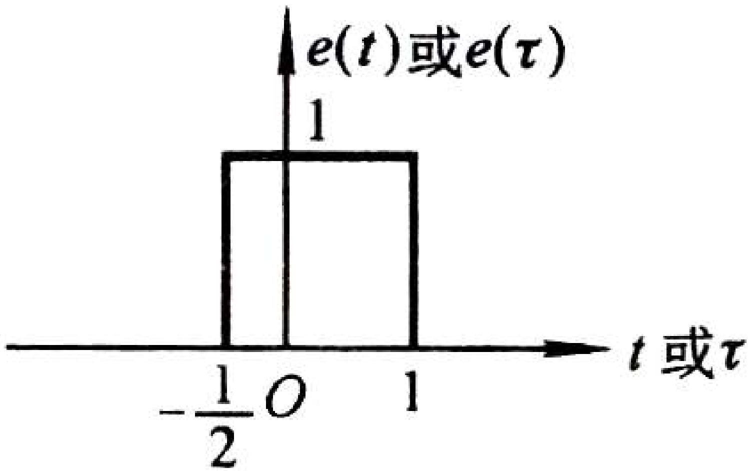


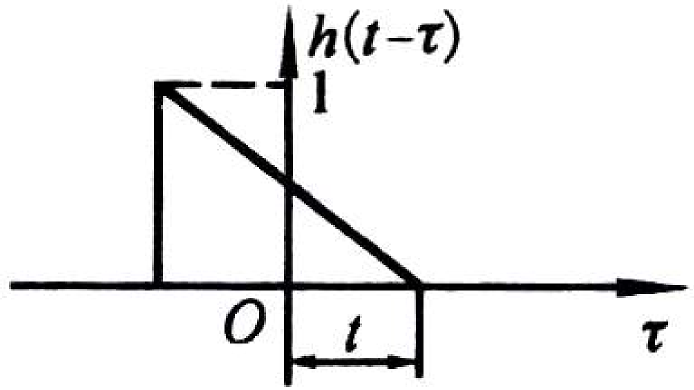
$$-1/2 \le \tau \le 1, 0 \le t - \tau \le 2$$

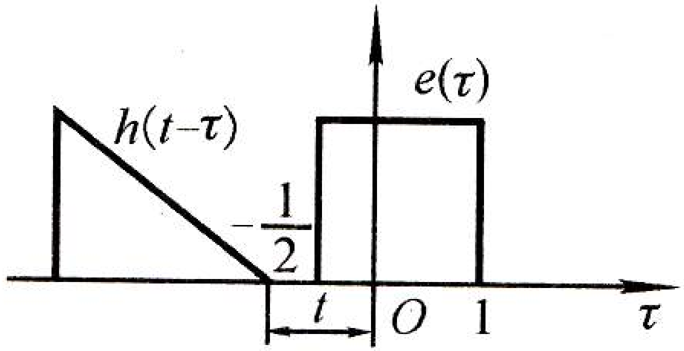
$$-1/2 \le t \le 3$$


h(t)或h(τ)
1
t 或τ
O
2

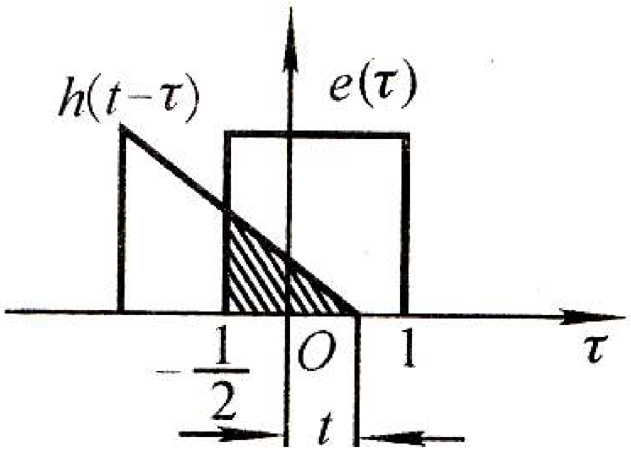

h(−τ)
1
−2
O
τ


e(t)或e(τ)
1
t 或τ
−1/2
O
1


h(t−τ)
1
O
t
τ



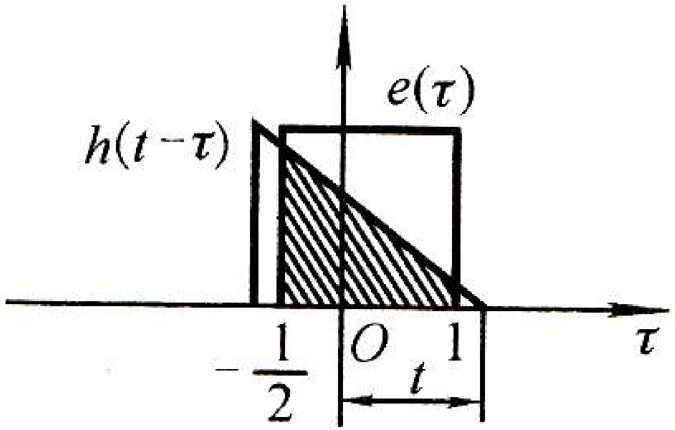
$$-\infty < t < -1/2$$



$$-1/2 < t < 1$$

$$s(t) = \int_{-1/2}^{t} 1 \times \frac{1}{2}(t-\tau)d\tau$$

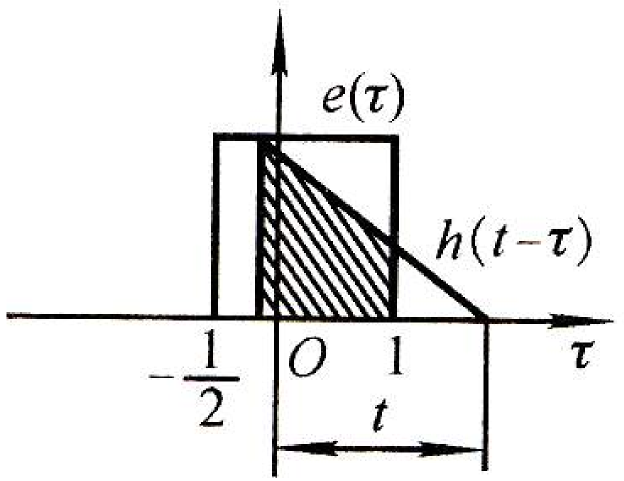
$$= \frac{t^2}{4} + \frac{t}{4} + \frac{1}{16}$$



$$1 < t < 3/2$$

$$s(t) = \int_{-1/2}^{1} 1 \times \frac{1}{2}(t-\tau)d\tau$$

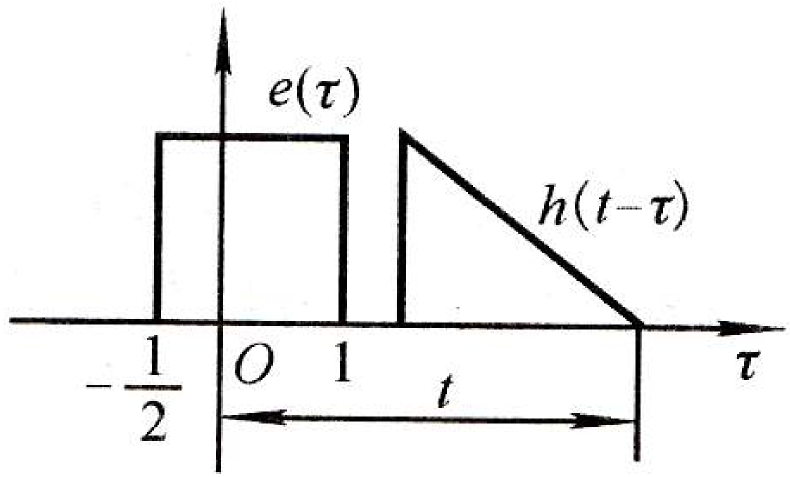
$$= \frac{3t}{4} - \frac{3}{16}$$



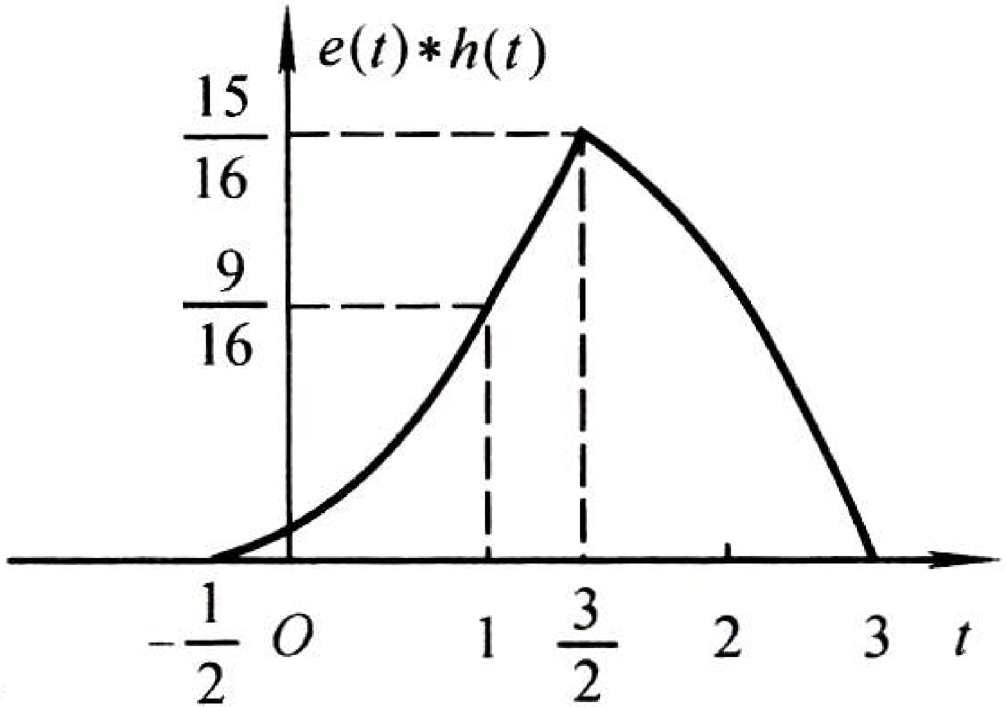
$3/2 < t < 3$

$$s(t) = \int_{t-2}^{1} 1 \times \frac{1}{2}(t-\tau)d\tau$$

$$= -\frac{t^2}{4} + \frac{t}{2} + \frac{3}{4}$$



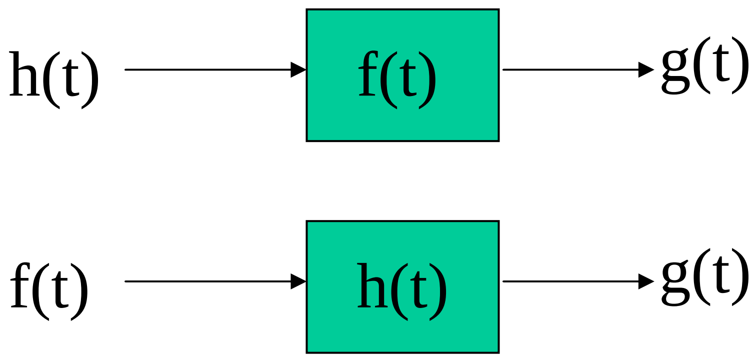
$3 < t < +\infty$



2.7　卷积的性质

- 交换律

$$f_1(t) * f_2(t) = f_2(t) * f_1(t)$$



$$s(t) = \int_{-\infty}^{\infty} f_1(\tau) f_2(t-\tau) d\tau$$

$$\lambda = t - \tau, d\lambda = -d\tau$$

$$s(t) = \int_{\infty}^{-\infty} f_1(t-\lambda) f_2(\lambda) - d\lambda$$

$$= \int_{-\infty}^{\infty} f_1(t-\lambda) f_2(\lambda) d\lambda = f_2(t) * f_1(t)$$

- 分配律

$$f_1(t)*[f_2(t)+f_3(t)]=f_1(t)*f_2(t)+f_1(t)*f_3(t)$$

  - 系统对几个相加输入信号的零状态响应等于分别对每个激励零状态响应的叠加

$$f_1(t)*[f_2(t)+f_3(t)]=\int_{-\infty}^{\infty} f_1(\tau)*[f_2(t-\tau)+f_3(t-\tau)]d\tau$$

$$=\int_{-\infty}^{\infty} f_1(\tau)*f_2(t-\tau)d\tau+\int_{-\infty}^{\infty} f_1(\tau)*f_3(t-\tau)d\tau$$

$$=f_1(t)*f_2(t)+f_1(t)*f_3(t)$$

- 结合律

$$[f_1(t)*f_2(t)]*f_3(t)=f_1(t)*[f_2(t)*f_3(t)]$$

$$[f_1(t)*f_2(t)]*f_3(t)=\int_{-\infty}^{\infty}[\int_{-\infty}^{\infty}f_1(\lambda)*f_2(\tau-\lambda)d\lambda]f_3(t-\tau)]d\tau$$

$$=\int_{-\infty}^{\infty}f_1(\lambda)[\int_{-\infty}^{\infty}f_2(\tau-\lambda)f_3(t-\tau)]d\tau d\lambda$$

$$\alpha=\tau-\lambda,\tau=\alpha+\lambda$$

$$=\int_{-\infty}^{\infty}f_1(\lambda)[\int_{-\infty}^{\infty}f_2(\alpha)f_3(t-\lambda-\alpha)]d\alpha d\lambda$$

$$=\int_{-\infty}^{\infty}f_1(\lambda)s_1(t-\lambda)d\lambda=f_1(t)*s_1(t)$$

- 微分和积分
  - 微分

$$\frac{d}{dt}[f_1(t) * f_2(t)] = [\frac{d}{dt} f_1(t)] * f_2(t) = [\frac{d}{dt} f_2(t)] * f_1(t)$$

$$f_1^{(n)}(t) * f_2(t) = f_1^{(n-k)}(t) * f_2^{(k)}(t)$$

  - 积分

$$\int_{-\infty}^{t} [f_1(\tau) * f_2(\tau)] d\tau = \int_{-\infty}^{t} f_1(\tau) d\tau * f_2(t) = \int_{-\infty}^{t} f_2(\tau) d\tau * f_1(t)$$

$$s^{(-n)}(t) = f_1^{(-n+k)}(t) * f_2^{(-k)}(t)$$

- 与冲激函数及阶跃函数的卷积
  - 与冲激函数卷积

$$f(t)*\delta(t)=f(t)$$

$$f(t)*\delta(t-t_0)=f(t_0)$$

$$f(t)*\delta'(t)=f'(t)$$

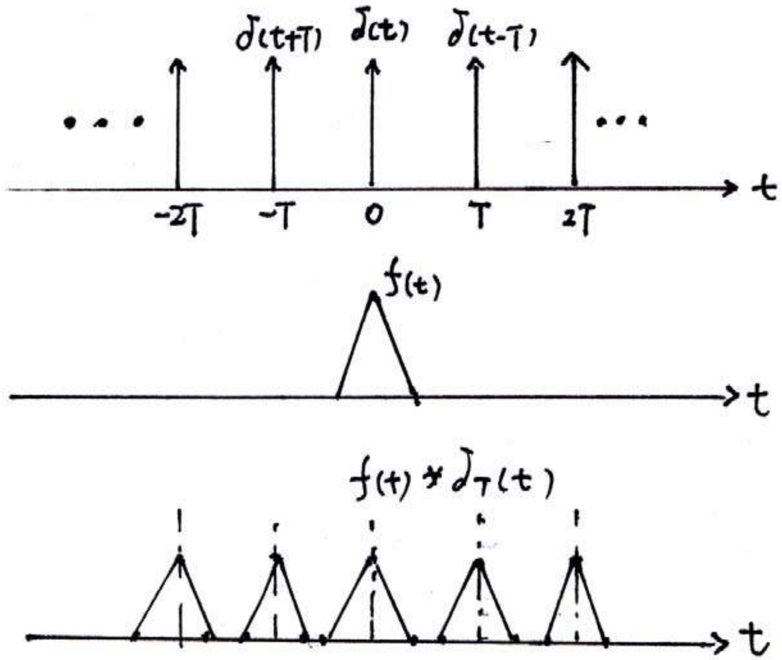
$$f(t)*\delta^{(k)}(t)=f^{(k)}(t)$$

  - 与阶跃函数卷积

$$f(t)*U(t)=\int_{-\infty}^{t}f(\tau)d\tau$$

- 例：冲激序列与f(t)的卷积

$$\delta_T(t) = \sum_{m=-\infty}^{\infty} \delta(t - mT)$$



$$f(t) * \delta_T(t) = f(t) * \sum_{m=-\infty}^{\infty} \delta(t - mT)$$

$$= \sum_{m=-\infty}^{\infty} f(t) * \delta(t - mT) = \sum_{m=-\infty}^{\infty} f(t - mT)$$

作　业

2-1 (b) (c) (d)

2-6

2-7

2-8

2-12

2-13 (3) (5)

2-15 (1) (4)

2-19 (c) (e)

2-20