Quantum Error Correction I

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Why Error Correction?

The Power of quantum computing:

$$U_f: \; |x\rangle|y\rangle \to |x\rangle|y \oplus f(x)\rangle$$

therefore

$$U_f \frac{1}{\sqrt{2^n}} \sum_{x=0}^{2^n-1} |x\rangle|0\rangle \to \frac{1}{\sqrt{2^n}} \sum_{x=0}^{2^n-1} |x\rangle|f(x)\rangle$$

Coherence – quantum parallelism!

Decoherence! $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \to \; |0\rangle$ or $|1\rangle$.

Schrodingers cat Coherence is ok for a few atoms or photons in lab when the coupling to environment is weak enough. For a system as big as a cat, comprised of billions upon billions of atoms, decoherence happens almost instantaneously, so that the cat can never be both alive and dead for any measurable instant.

Why Error Correction?

Reading: by Serge Haroche and Jean-Michel Raimond, Physics Today, 51, August 1996

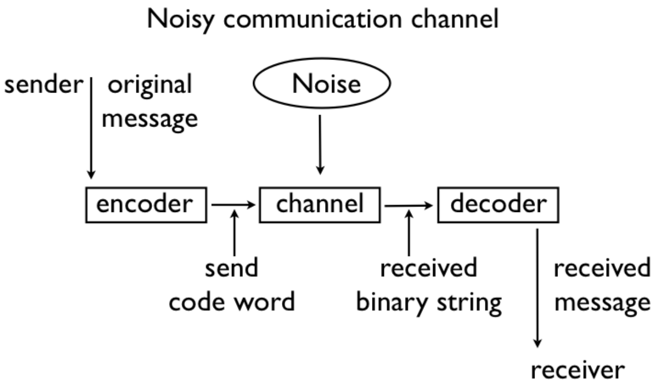
Peter Shor: There is no real hard problem in the world...We are simply not smart enough...

	Analog Computer	Quantum Computer
Input	$\vec{x}(0)$	$\vert\psi\rangle$
Computing	$\frac{d\vec{x}}{dt} = f(\vec{x})$	$i\frac{\partial}{\partial t}\vert\psi\rangle = \mathcal{H}\vert\psi\rangle$
Output	$\vec{x}(T)$	measurement

Analog Computers are Continuous, Unreliable. They have been replaced by digital computers for almost all uses.

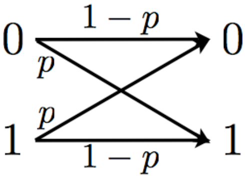
Can we build a DIGITAL quantum computer?

Basic ideas for Error Correction



Classical Error Correction

Digital Communication.



Binary Symmetric Channel.
The Repetition Code:

$$0 \to 000, \quad 1 \to 111.$$

Decoding: Majority Voting.

Probability of Two flips: $3p^2(1-p) + p^3$, therefore probability of error: $p_e = 3p^2 - 2p^3$. $p_e < p$ if $p < \frac{1}{2}$.

Classical Error Correction

In Classical World... Digital computer with error correction:

$$0 \to 000, \quad 1 \to 111.$$

Errors $0 \leftrightarrow 1$, discrete.

But in Quantum World...

◇ Continuous errors:

$$|0\rangle \to U|0\rangle, \ |1\rangle \to U|1\rangle$$

◇ Measurement destroys coherence!

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

◇ No-cloning theorem!

$$|\psi\rangle|\psi\rangle \neq \alpha|0\rangle|0\rangle + \beta|1\rangle|1\rangle$$

Still hopeless?

Starting from a Simple Case

Binary Symmetric Channel: $0 \leftrightarrow 1$.

Quantum Bit flip Channel: $|0\rangle \leftrightarrow |1\rangle$. $\mathbf{X} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

The Repetition Code:

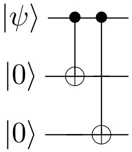
$$0 \rightarrow 000, \quad 1 \rightarrow 111.$$

The Quantum Bit Flip Code:

$$|0\rangle \rightarrow |000\rangle \equiv |0_L\rangle, \quad |1\rangle \rightarrow |111\rangle \equiv |1_L\rangle$$

$$|\psi\rangle = a|0\rangle + b|1\rangle \rightarrow a|000\rangle + b|111\rangle$$

Encoding Circuit:



Quantum Bit Flip Code

$$|\psi\rangle = a|0\rangle + b|1\rangle \rightarrow a|000\rangle + b|111\rangle$$

Bit Flip Errors:

$$\begin{aligned} |\psi_0\rangle &= a|000\rangle + b|111\rangle && \text{No Error} \\ |\psi_1\rangle &= a|100\rangle + b|011\rangle && \text{flip on the 1st qubit} \\ |\psi_2\rangle &= a|010\rangle + b|101\rangle && \text{flip on the 2nd qubit} \\ |\psi_3\rangle &= a|001\rangle + b|110\rangle && \text{flip on the 3rd qubit} \end{aligned}$$

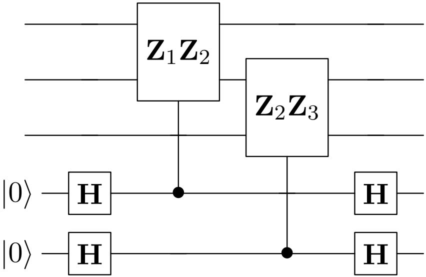
Syndrome Diagnosis:

$$\begin{aligned} P_0 &\equiv |000\rangle\langle 000| + |111\rangle\langle 111| && \text{No Error} \\ P_1 &\equiv |100\rangle\langle 100| + |011\rangle\langle 011| && \text{flip on the 1st qubit} \\ P_2 &\equiv |010\rangle\langle 010| + |101\rangle\langle 101| && \text{flip on the 2nd qubit} \\ P_3 &\equiv |001\rangle\langle 001| + |110\rangle\langle 110| && \text{flip on the 3rd qubit} \end{aligned}$$

Quantum Bit Flip Code

Syndrome Measurements:

	$\mathbf{Z}_1\mathbf{Z}_2$	$\mathbf{Z}_2\mathbf{Z}_3$	Recovery
$\lvert\psi_0\rangle = a\lvert 000\rangle + b\lvert 111\rangle$	0	0	$\mathbf{I}$
$\lvert\psi_1\rangle = a\lvert 100\rangle + b\lvert 011\rangle$	1	0	$\mathbf{X}_1$
$\lvert\psi_2\rangle = a\lvert 010\rangle + b\lvert 101\rangle$	1	1	$\mathbf{X}_2$
$\lvert\psi_3\rangle = a\lvert 001\rangle + b\lvert 110\rangle$	0	1	$\mathbf{X}_3$



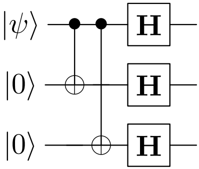
The Phase Flip Code

The Quantum Phase Flip Channel

$$|0\rangle \rightarrow |0\rangle \quad |1\rangle \rightarrow -|1\rangle, \quad \mathbf{Z} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Recall the Hadamard Transform $\mathbf{H} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$, and $\mathbf{HXH} = \mathbf{Z}$, which transforms $|0\rangle \rightarrow |+\rangle, \ |1\rangle \rightarrow |-\rangle$. Therefore we can do the encoding

$$|0\rangle \rightarrow |+++\rangle \equiv |0_L\rangle, \quad |1\rangle \rightarrow |---\rangle \equiv |1_L\rangle$$



Combination of Errors

Theorem

If a Quantum Code corrects errors $\mathbf{A}$ *and* $\mathbf{B}$*, it also corrects* $\alpha\mathbf{A} + \beta\mathbf{B}$.

Example

For the bit flip channel $\mathbf{X} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, consider the error $\alpha\mathbf{I_1} + \beta\mathbf{X_1}$. Use the encoding

$$|\psi\rangle = a|0\rangle + b|1\rangle \rightarrow a|000\rangle + b|111\rangle$$

the output will be $\alpha|\psi_0\rangle + \beta|\psi_1\rangle$, with

$$|\psi_0\rangle = a|000\rangle + b|111\rangle, \quad |\psi_1\rangle = a|100\rangle + b|011\rangle$$

We can still use the syndrome measurements $\mathbf{Z}_1\mathbf{Z}_2$ and $\mathbf{Z}_2\mathbf{Z}_3$.

An arbitrary single-qubit error: a linear combination of $\mathbf{I}, \mathbf{X}, \mathbf{Y}, \mathbf{Z}$.

Shor Code

Bit Flip: $|0\rangle \to |000\rangle, |1\rangle \to |111\rangle$.
Phase Flip: $|0\rangle \to |+++\rangle$, $|1\rangle \to |---\rangle$.
And

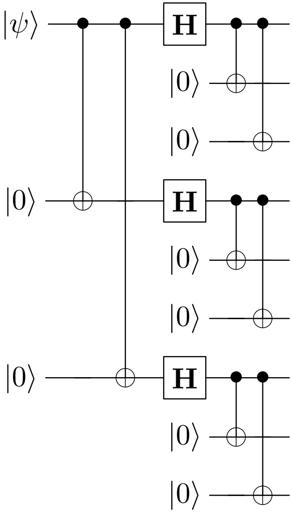
$$
\begin{aligned}
|+\rangle &= \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \to \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle) \\
|-\rangle &= \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \to \frac{1}{\sqrt{2}}(|000\rangle - |111\rangle)
\end{aligned}
$$

To fight against both bit flip and phase flip errors, we do a two step encoding: first encode to a phase flip code, then further encode to a bit flip code, we get the Shor Code

$$
\begin{aligned}
|0_L\rangle &\to \tfrac{1}{2\sqrt{2}}(|000\rangle + |111\rangle)(|000\rangle + |111\rangle)(|000\rangle + |111\rangle) \\
|1_L\rangle &\to \tfrac{1}{2\sqrt{2}}(|000\rangle - |111\rangle)(|000\rangle - |111\rangle)(|000\rangle - |111\rangle)
\end{aligned}
$$

Shor Code

Encoding circuit:



Syndrome measurements:
Bit Flip:

$$\mathbf{Z}_1\mathbf{Z}_2, \quad \mathbf{Z}_2\mathbf{Z}_3$$
$$\mathbf{Z}_4\mathbf{Z}_5, \quad \mathbf{Z}_5\mathbf{Z}_6$$
$$\mathbf{Z}_7\mathbf{Z}_8, \quad \mathbf{Z}_7\mathbf{Z}_9$$

Phase Flip:

$$\mathbf{X}_1\mathbf{X}_2\mathbf{X}_3\mathbf{X}_4\mathbf{X}_5\mathbf{X}_6$$
$$\mathbf{X}_4\mathbf{X}_5\mathbf{X}_6\mathbf{X}_7\mathbf{X}_8\mathbf{X}_9$$

Recovery:
$\mathbf{Z}_i$ for phase flip,
$\mathbf{X}_i$ for bit flip.

Commuting Pauli Operators

$$\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \; \mathbf{X} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \; \mathbf{Y} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \; \mathbf{Z} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

The commutation relations:

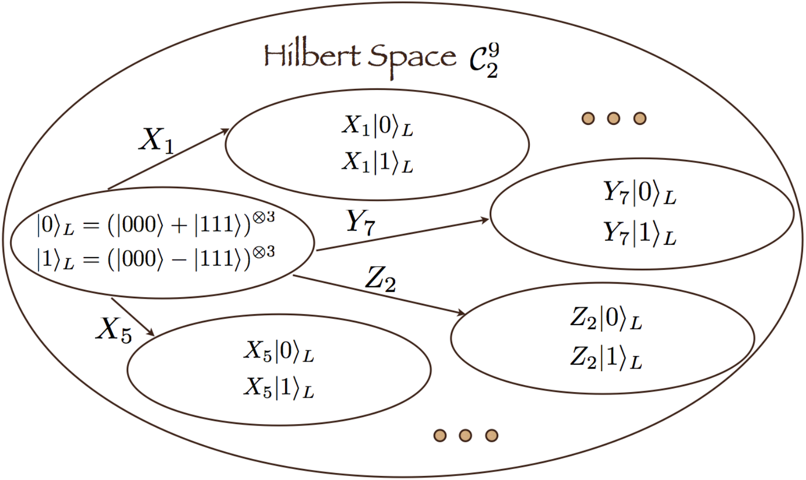
$$\mathbf{XY} = -\mathbf{YX}, \; \mathbf{XZ} = -\mathbf{ZX}, \; \mathbf{YZ} = -\mathbf{ZY}$$

Shor Code:

$$\begin{array}{ccccccccc}
\mathbf{Z} & \mathbf{Z} & \mathbf{I} & \mathbf{I} & \mathbf{I} & \mathbf{I} & \mathbf{I} & \mathbf{I} & \mathbf{I} \\
\mathbf{I} & \mathbf{Z} & \mathbf{Z} & \mathbf{I} & \mathbf{I} & \mathbf{I} & \mathbf{I} & \mathbf{I} & \mathbf{I} \\
\mathbf{I} & \mathbf{I} & \mathbf{I} & \mathbf{Z} & \mathbf{Z} & \mathbf{I} & \mathbf{I} & \mathbf{I} & \mathbf{I} \\
\mathbf{I} & \mathbf{I} & \mathbf{I} & \mathbf{I} & \mathbf{Z} & \mathbf{Z} & \mathbf{I} & \mathbf{I} & \mathbf{I} \\
\mathbf{I} & \mathbf{I} & \mathbf{I} & \mathbf{I} & \mathbf{I} & \mathbf{I} & \mathbf{Z} & \mathbf{Z} & \mathbf{I} \\
\mathbf{I} & \mathbf{I} & \mathbf{I} & \mathbf{I} & \mathbf{I} & \mathbf{I} & \mathbf{I} & \mathbf{Z} & \mathbf{Z} \\
\mathbf{X} & \mathbf{X} & \mathbf{X} & \mathbf{X} & \mathbf{X} & \mathbf{X} & \mathbf{I} & \mathbf{I} & \mathbf{I} \\
\mathbf{I} & \mathbf{I} & \mathbf{I} & \mathbf{X} & \mathbf{X} & \mathbf{X} & \mathbf{X} & \mathbf{X} & \mathbf{X}
\end{array}$$

$$\begin{aligned}
|0_L\rangle &\rightarrow \frac{1}{2\sqrt{2}}(|000\rangle + |111\rangle)(|000\rangle + |111\rangle)(|000\rangle + |111\rangle) \\
|1_L\rangle &\rightarrow \frac{1}{2\sqrt{2}}(|000\rangle - |111\rangle)(|000\rangle - |111\rangle)(|000\rangle - |111\rangle)
\end{aligned}$$

A Picture of Shor's code



Quantum Error Correcting Criterion

Quantum Code: A subspace of $\mathbb{C}_2^{\otimes n}$

Code space basis $\{|\psi_i\rangle\}$

Errors: $\{\mathbf{E}_\alpha\}$

Quantum Error Correcting Criterion:

$$\langle\psi_i|\mathbf{E}_\alpha^\dagger \mathbf{E}_\beta|\psi_j\rangle = c_{\alpha\beta}\delta_{ij}$$

◇ Orthogonal Condition: $\mathbf{E}_\alpha|\psi_i\rangle \perp \mathbf{E}_\beta|\psi_j\rangle$.

Classical: $i \neq j$.

◇ Coherence Condition: $\langle\psi_i|\mathbf{E}_\alpha^\dagger \mathbf{E}_\beta|\psi_i\rangle = c_{\alpha\beta}$.

Quantum: $i = j$.

Hamming Code

Recall that the Repetition Code, with encoding
$0 \to 000,\ 1 \to 111$, we write the Code Parameters for this code as $[3, 1]$.
This code corrects $t = 1$ error. Define the Code Distance $d = 2t + 1$. We write it as $[3, 1, 3]$.
In general, for an $[n, k, d]$ code, we would want n small, k large and d large. But there are certainly trade-offs.
Let us first fix $d = 3$, and want a large rate k/n. We will show the construction of the $[7, 4, 3]$ Hamming Code.
We start from the following encoding to make a Linear Code

$$\begin{aligned} 1000 &\to 1000110 \\ 0100 &\to 0100101 \\ 0010 &\to 0010011 \\ 0001 &\to 0001111 \end{aligned}$$

Hamming Code

We can write a Generator Matrix

$$\mathbf{G} = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

then the encoding becomes

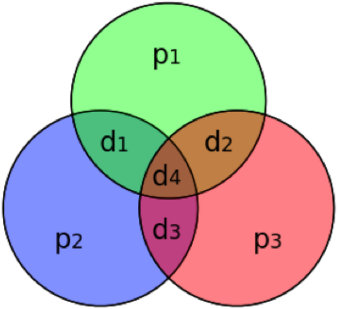
$$\mathbf{x} = \mathbf{aG}$$

where

$$\mathbf{a} = a_3 a_2 a_1 a_0,$$

and

$$\begin{aligned} \mathbf{x} &= x_6 x_5 x_4 x_3 x_2 x_1 x_0 \\ & \quad d_1 d_2 d_3 d_4 p_1 p_2 p_3 \end{aligned}$$



Syndrome: p_1, p_2, p_3.

Quantum 7-bit Code

Hamming code

$$\mathbf{G} = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Write its even subcode

$$\mathbf{G_e} = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

We now build a quantum 7-bit code via the encoding

$$\begin{aligned} |0\rangle \to |0\rangle_L &= \frac{1}{2\sqrt{2}} \sum_{\mathbf{x}\in\mathbf{G_e}} |\mathbf{x}\rangle \\ &= |0000000\rangle + |1100011\rangle + |0110110\rangle + |0001111\rangle \\ &+ |1010101\rangle + |1101100\rangle + |0111001\rangle + |1011010\rangle \\ |1\rangle \to |1\rangle_L &= \frac{1}{2\sqrt{2}} \sum_{\mathbf{x}\in\mathbf{G}\setminus\mathbf{G_e}} |\mathbf{x}\rangle \\ &= |1111111\rangle + |0011100\rangle + |1001001\rangle + |1110000\rangle \\ &+ |0101010\rangle + |0010011\rangle + |1000110\rangle + |0100101\rangle \end{aligned}$$

Syndrome Measurements

For

$$\mathbf{G_e} = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Let

$$\begin{aligned} \mathbf{M}_1 &= \mathbf{X}_1\mathbf{X}_2\mathbf{X}_6\mathbf{X}_7 \\ \mathbf{M}_2 &= \mathbf{X}_2\mathbf{X}_3\mathbf{X}_5\mathbf{X}_6 \\ \mathbf{M}_3 &= \mathbf{X}_4\mathbf{X}_5\mathbf{X}_6\mathbf{X}_7 \end{aligned}$$

Then we can write

$$|0_L\rangle = \frac{1}{2\sqrt{2}} \sum_{\mathbf{x}\in\mathbf{G_e}} |\mathbf{x}\rangle = (\mathbf{I}+\mathbf{M}_1)(\mathbf{I}+\mathbf{M}_2)(\mathbf{I}+\mathbf{M}_3)|0\rangle_7$$

$$|1_L\rangle = \frac{1}{2\sqrt{2}} \sum_{\mathbf{x}\in\mathbf{G}\setminus\mathbf{G_e}} |\mathbf{x}\rangle = (\mathbf{I}+\mathbf{M}_1)(\mathbf{I}+\mathbf{M}_2)(\mathbf{I}+\mathbf{M}_3)\mathbf{X}_L|0\rangle_7$$

where

$$\mathbf{X}_L = \mathbf{X}_1\mathbf{X}_2\mathbf{X}_3\mathbf{X}_4\mathbf{X}_5\mathbf{X}_6\mathbf{X}_7$$

Phase flip syndromes: $\mathbf{M}_1, \mathbf{M}_2, \mathbf{M}_3$.

Syndrome Measurements

$$\begin{aligned} \mathbf{M}_1 &= \mathbf{X}_1\mathbf{X}_2\mathbf{X}_6\mathbf{X}_7 & \mathbf{N}_1 &= \mathbf{Z}_1\mathbf{Z}_2\mathbf{Z}_6\mathbf{Z}_7 \\ \mathbf{M}_2 &= \mathbf{X}_2\mathbf{X}_3\mathbf{X}_5\mathbf{X}_6 & \mathbf{N}_2 &= \mathbf{Z}_2\mathbf{Z}_3\mathbf{Z}_5\mathbf{Z}_6 \\ \mathbf{M}_3 &= \mathbf{X}_4\mathbf{X}_5\mathbf{X}_6\mathbf{X}_7 & \mathbf{N}_3 &= \mathbf{Z}_4\mathbf{Z}_5\mathbf{Z}_6\mathbf{Z}_7 \end{aligned}$$

Then for

$$|0_L\rangle = (\mathbf{I}+\mathbf{M}_1)(\mathbf{I}+\mathbf{M}_2)(\mathbf{I}+\mathbf{M}_3)|0\rangle_7$$
$$|1_L\rangle = (\mathbf{I}+\mathbf{M}_1)(\mathbf{I}+\mathbf{M}_2)(\mathbf{I}+\mathbf{M}_3)\mathbf{X}_L|0\rangle_7$$

Bit flip syndromes: $\mathbf{N}_1, \mathbf{N}_2, \mathbf{N}_3$.
Logic Operations: (Fault-Tolerance)

$$\begin{aligned} \mathbf{X}_L &= \mathbf{X}_1\mathbf{X}_2\mathbf{X}_3\mathbf{X}_4\mathbf{X}_5\mathbf{X}_6\mathbf{X}_7 \\ \mathbf{Z}_L &= \mathbf{Z}_1\mathbf{Z}_2\mathbf{Z}_3\mathbf{Z}_4\mathbf{Z}_5\mathbf{Z}_6\mathbf{Z}_7 \\ \mathbf{H}_L &= \mathbf{H}_1\mathbf{H}_2\mathbf{H}_3\mathbf{H}_4\mathbf{H}_5\mathbf{H}_6\mathbf{H}_7 \\ \mathbf{CNOT}_L &= \mathbf{C}_{1,8}\mathbf{C}_{2,9}\mathbf{C}_{3,10}\mathbf{C}_{4,11}\mathbf{C}_{5,12}\mathbf{C}_{6,13}\mathbf{C}_{7,14} \end{aligned}$$

Five Qubit Code

For the parameters $[5, 1, 3]$, we have a perfect code because $(1 + (3 \times 5)) \times 2 = 2^5$. Let

$$
\begin{array}{rcccccc}
\mathbf{M}_1 & = & \mathbf{Z} & \mathbf{X} & \mathbf{X} & \mathbf{Z} & \mathbf{I} \\
\mathbf{M}_2 & = & \mathbf{I} & \mathbf{Z} & \mathbf{X} & \mathbf{X} & \mathbf{Z} \\
\mathbf{M}_3 & = & \mathbf{Z} & \mathbf{I} & \mathbf{Z} & \mathbf{X} & \mathbf{X} \\
\mathbf{M}_4 & = & \mathbf{X} & \mathbf{Z} & \mathbf{I} & \mathbf{Z} & \mathbf{X} \\
\mathbf{X}_L & = & \mathbf{X} & \mathbf{X} & \mathbf{X} & \mathbf{X} & \mathbf{X} \\
\mathbf{Z}_L & = & \mathbf{Z} & \mathbf{Z} & \mathbf{Z} & \mathbf{Z} & \mathbf{Z}
\end{array}
$$

we do the encoding

$$
\begin{aligned}
|0_L\rangle &= \frac{1}{4}(\mathbf{I} + \mathbf{M}_1)(\mathbf{I} + \mathbf{M}_2)(\mathbf{I} + \mathbf{M}_3)(\mathbf{I} + \mathbf{M}_4)|0\rangle_5 \\
|1_L\rangle &= \frac{1}{4}(\mathbf{I} + \mathbf{M}_1)(\mathbf{I} + \mathbf{M}_2)(\mathbf{I} + \mathbf{M}_3)(\mathbf{I} + \mathbf{M}_4)\mathbf{X}_L|0\rangle_5
\end{aligned}
$$

Note that $\mathbf{M}_i^2 = 1$ and $(\mathbf{I} + \mathbf{M}_i)^2 = \mathbf{I} + \mathbf{M}_i$ and

$$(\mathbf{I} + \mathbf{M}_i)(\mathbf{I} - \mathbf{M}_i) = 0$$

so it is straightforward to show that

$$\langle 0_L | 1_L \rangle = 0,$$

and further the quantum error correcting criterion, e.g.

$$\langle 0_L | \mathbf{X}_1 \mathbf{Y_2} | 1_L \rangle = 0, \ \langle 0_L | \mathbf{X}_1 \mathbf{Y_2} | 0_L \rangle = 0, \ \langle 1_L | \mathbf{X}_1 \mathbf{Y_2} | 1_L \rangle = 0$$

Syndrome measurements:

	$\mathbf{I}$	$\mathbf{X}_1$	$\mathbf{Y}_1$	$\mathbf{Z}_1$	$\mathbf{X}_2$	$\mathbf{Y}_2$	$\mathbf{Z}_2$	$\mathbf{X}_3$	$\mathbf{Y}_3$	$\mathbf{Z}_3$	$\mathbf{X}_4$	$\mathbf{Y}_4$	$\mathbf{Z}_4$	$\mathbf{X}_5$	$\mathbf{Y}_5$	$\mathbf{Z}_5$
$\mathbf{M}_1$	0	1	1	0	0	1	1	0	1	1	1	1	0	0	0	0
$\mathbf{M}_2$	0	0	0	0	1	1	0	0	1	1	0	1	1	1	1	0
$\mathbf{M}_3$	0	1	1	0	0	0	0	1	1	0	0	1	1	0	1	1
$\mathbf{M}_4$	0	0	1	1	1	1	0	0	0	0	1	1	0	0	1	1

Threshold Theorem

Error Correcting Code: $p \to Cp^2$

Concatenation of Codes:

Twice: error probability	$C(Cp^2)^2$	
k times: error probability	$C(Cp^2)^2$	doubly exponential
size of the circuit	d^k	exponential

Threshold Theorem

An arbitrary long quantum computation can be performed reliably, provided that the average probability of error per gate is less than a certain critical value, the accuracy threshold.

Note: The accuracy threshold depends on quantum code ALONE!

Threshold Theorem

So....are we below threshold?

◇ Perhaps NOT: $p \sim 10^{-5}$, orders of magnitude away....

◇ We are...BELOW threshold! – Recent advances combining physics and computer science: Quantum computing against biased noise http://arxiv.org/abs/0710.1301

◇ Should we celebrate? Perhaps NO – we are JUST below threshold overhead are large...

Both threshold and overhead depend on quantum code ALONE!

◇ Yes? Making BETTER quantum codes! Better quantum codes can be designed. We are full of hope, when computer scientists meeting with physicists...