Deep Learning Normalization

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Batch Normalization [Ioffe, Szegedy]

For a layer of input vector x:

Normalize:

$$\widehat{x}^{(k)} = \frac{x^{(k)} - \mathrm{E}[x^{(k)}]}{\sqrt{\mathrm{Var}[x^{(k)}]}}$$

$$y^{(k)} = \gamma^{(k)}\widehat{x}^{(k)} + \beta^{(k)}$$

Additional parameters to learn (thru BP)

- BP needs to be modified to account for the change 反向传播过程需要修正
- Improves gradient flow through the network 可以改善网络的梯度流
- Allows higher learning rates 允许更高的学习速率
- Reduces the strong dependence on initialization 减少对初始化的依赖
- Acts as a form of regularization in a funny way, and slightly reduces the need for dropout, maybe 在某种程度上是正则化

Tensorflow Code:
tf.nn. batch_normalization(x, mean, variance, offset, scale, variance_epsilon, name=None)
Keras Code:
keras.layers.normalization.BatchNormalization(axis=-1, momentum=0.99, epsilon=0.001, center=**True**, scale=**True**)

Input: Values of x over a mini-batch: $\mathcal{B} = \{x_{1...m}\}$;
Parameters to be learned: γ, β

Output: $\{y_i = \mathrm{BN}_{\gamma,\beta}(x_i)\}$

$$\mu_{\mathcal{B}} \leftarrow \frac{1}{m}\sum_{i=1}^{m} x_i \qquad \text{// mini-batch mean}$$

$$\sigma^2_{\mathcal{B}} \leftarrow \frac{1}{m}\sum_{i=1}^{m}(x_i - \mu_{\mathcal{B}})^2 \qquad \text{// mini-batch variance}$$

$$\widehat{x}_i \leftarrow \frac{x_i - \mu_{\mathcal{B}}}{\sqrt{\sigma^2_{\mathcal{B}} + \epsilon}} \qquad \text{// normalize}$$

$$y_i \leftarrow \gamma\widehat{x}_i + \beta \equiv \mathrm{BN}_{\gamma,\beta}(x_i) \qquad \text{// scale and shift}$$

Algorithm 1: Batch Normalizing Transform, applied to activation x over a mini-batch.

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Why BN??

- Internal Covariate Shift
- In statistical learning, one typically assumes that the source distribution (training) and the target distribution (testing) are the same

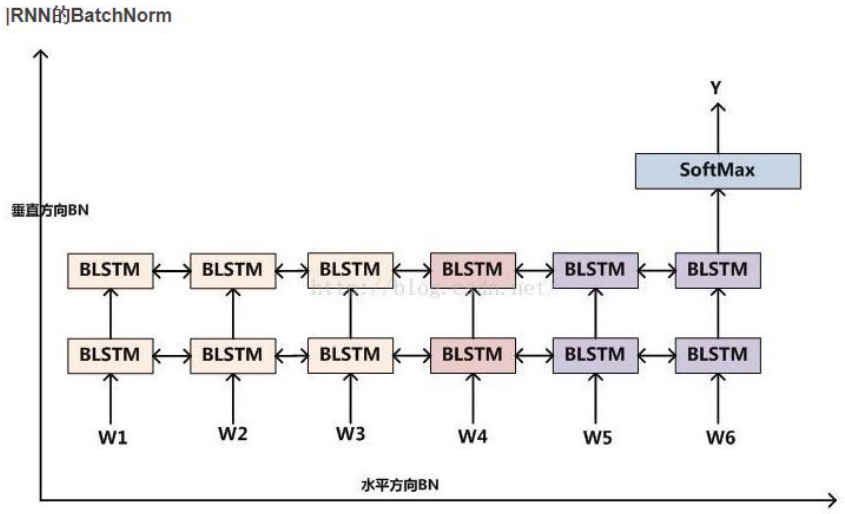
  (if not, consider transfer learning)

Covariate shift: $x \in \mathcal{X}, P_s(Y|X=x) = P_t(Y|X=x)$

$$P_s(X) \neq P_t(X)$$

In neural networks, after a few layers' transformations, the distribution may change a lot.

- Batch Normalization relates to mini-batch size
- Typically, BN requires larger mini-batch size
- Not easy to applied to RNN (possible, but no conclusion)



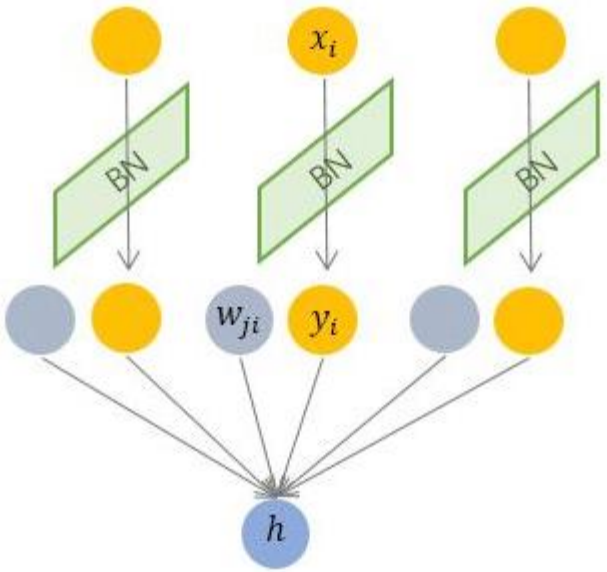
Previous attempt to do BN on RNN

$$\mathbf{h}_t = \phi(BN(\mathbf{W}_h\mathbf{h}_{t-1} + \mathbf{W}_x\mathbf{x}_t))$$

BN for both horizontal and vertical directions.
Doesn't work
Horizontal: may need BN with different parameters for different time steps

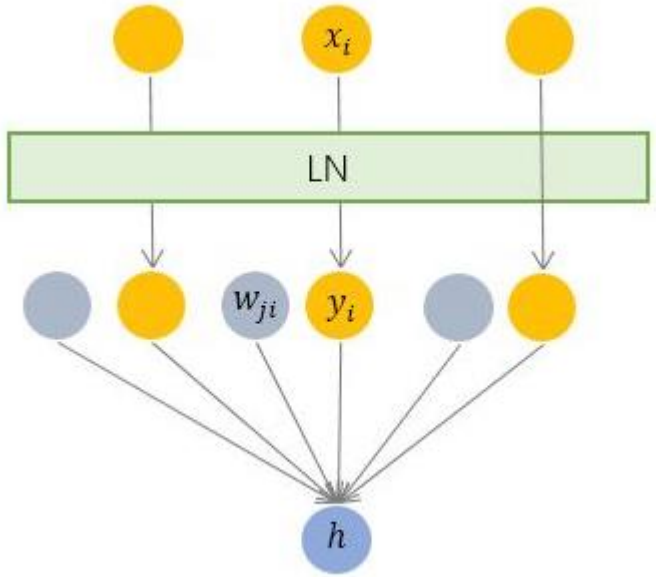
Layer Normalization

BN



Normalization over a mini-batch

LN



Normalization over a layer
(for a single sample)

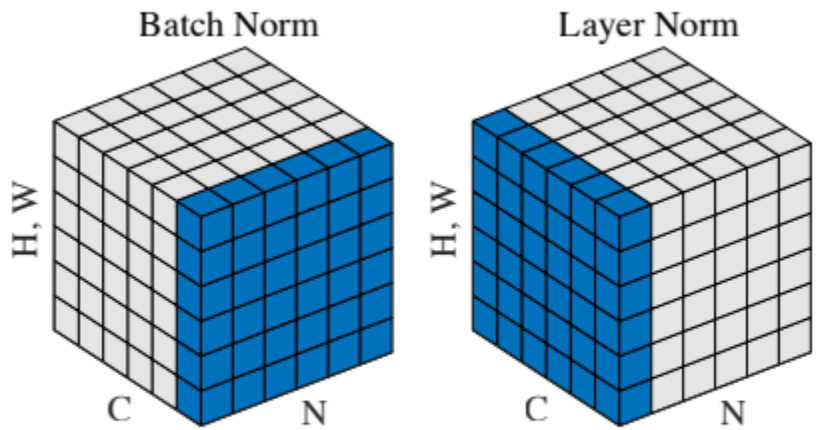


Figure 2. **Normalization methods**. Each subplot shows a feature map tensor, with N as the batch axis, C as the channel axis, and (H, W) as the spatial axes. The pixels in blue are normalized by the same mean and variance, computed by aggregating the values of these pixels.

Some details

- Normalization $\bar{a}_i^l = \frac{g_i^l}{\sigma_i^l}(a_i^l - \mu_i^l)$

gi: a gain parameter scaling the normalized activation before the non-linear activation function

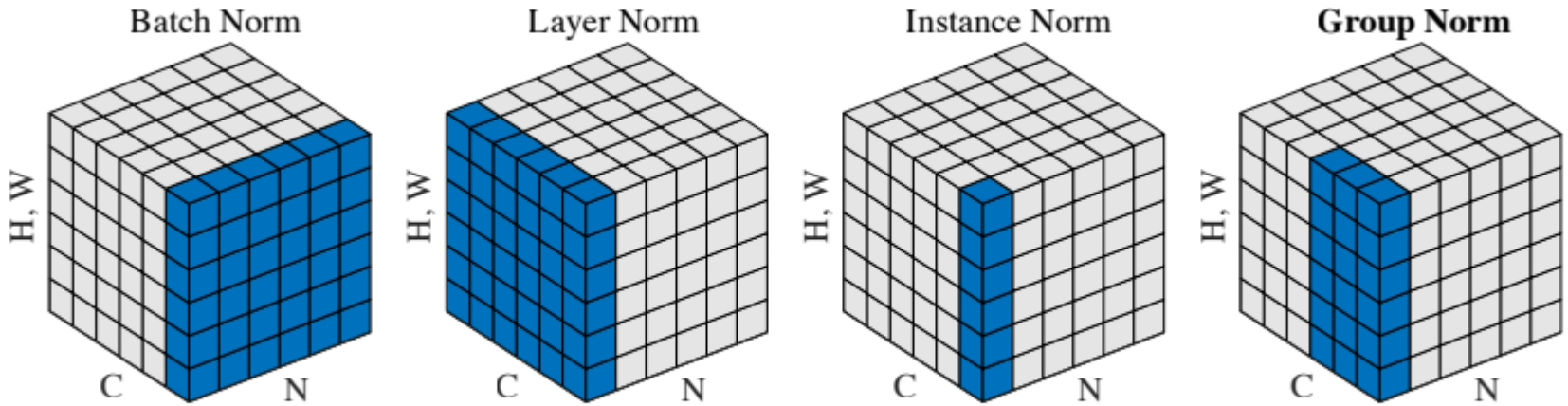
- For a feedforward network, sum over a layer
- In CNN, summation over (C,H,W)

$$\mu^l = \frac{1}{H}\sum_{i=1}^{H} a_i^l \qquad \sigma^l = \sqrt{\frac{1}{H}\sum_{i=1}^{H}(a_i^l - \mu^l)^2}$$

- In RNN, sum over a layer $\mathbf{a}^t = W_{hh}h^{t-1} + W_{xh}\mathbf{x}^t$

$$\mathbf{h}^t = f\left[\frac{\mathbf{g}}{\sigma^t} \odot (\mathbf{a}^t - \mu^t) + \mathbf{b}\right] \qquad \mu^t = \frac{1}{H}\sum_{i=1}^{H} a_i^t \qquad \sigma^t = \sqrt{\frac{1}{H}\sum_{i=1}^{H}(a_i^t - \mu^t)^2}$$

Group Normalization



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Batch Norm:
+ Stable if the batch size is large
+ Robust (in train) to the scale & shift of input data
+ Robust to the scale of weight vector
+ Scale of update decreases while training
- Not good for online learning
- Not good for RNN, LSTM
- Different calculation between train and test

Layer Norm:
+ Effective to small mini batch RNN
+ Robust to the scale of input
+ Robust to the scale and shift of weight matrix
+ Scale of update decreases while training
- Might be not good at CNN?
(Batch Norm is better in some cases)

- Other normalizations: weight normalization

Thanks