§4 相对论理论的四维形式

❖ 相对论中的时间、空间是紧密相联系的；

❖ 三维空间和一维时间构成一个统一的整体

——**四维时空**

本节的主要内容：

一．回顾：三维空间的正交变换

二．物理量按三维空间变换性质的分类

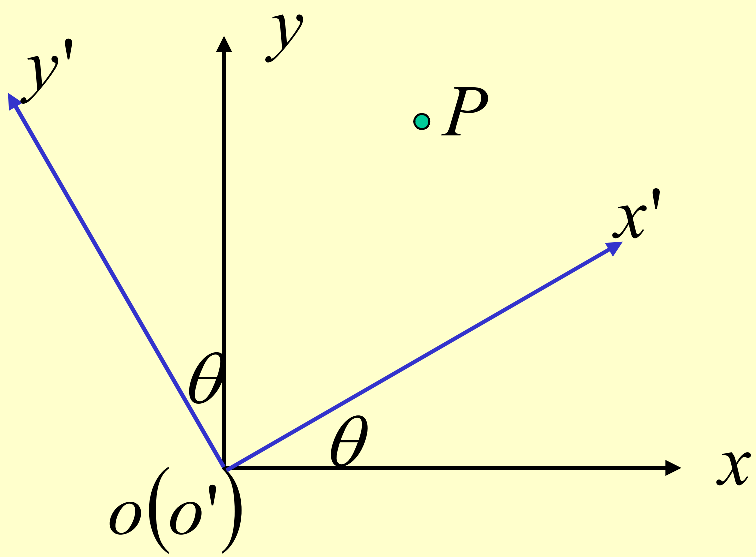
三．Lorentz变换的四维形式

四．四维协变量

五．多普勒效应和光行差

一、三维空间的正交变换

1、平面上坐标系的转动



平面上的一点 P

$$\Sigma:(x,\ y);\quad \Sigma':(x',\ y')$$

变换关系：

$$x' = x\cos\theta + y\sin\theta,$$
$$y' = -x\sin\theta + y\cos\theta$$

P点到坐标原点（假设重合）的距离保持不变：

$$x^2 + y^2 = x'^2 + y'^2$$

2、在三维坐标系发生转动下

空间一点 P： $\Sigma:(x_1, x_2, x_3);$

$$\Sigma':(x_1', x_2', x_3')$$

变换关系为：

$$\begin{cases} x_1' = a_{11}x_1 + a_{12}x_2 + a_{13}x_3 \\ x_2' = a_{21}x_1 + a_{22}x_2 + a_{23}x_3 \\ x_3' = a_{31}x_1 + a_{32}x_2 + a_{33}x_3 \end{cases}$$

$$\begin{cases} x_1' = a_{11}x_1 + a_{12}x_2 + a_{13}x_3 \\ x_2' = a_{21}x_1 + a_{22}x_2 + a_{23}x_3 \\ x_3' = a_{31}x_1 + a_{32}x_2 + a_{33}x_3 \end{cases}$$

矩阵形式为：

$$\begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

一般地写成： $x_i' = \sum_{j=1}^{3} a_{ij} x_j, \quad (i = 1,\ 2,\ 3)$

$$\boxed{x_i' = a_{ij} x_j, \quad (i = 1,\ 2,\ 3)}$$

（一项中重复出现的下标则隐含有爱因斯坦求和规则）

坐标系在转动时，P点与坐标原点的距离保持不变，

$$\sum_{i=1}^{3} x_i'^2 = \sum_{i=1}^{3} x_i^2$$

① 满足上述条件的线性变换称为正交变换；

② 上述条件也称为正交条件。

$$x_i' = a_{ij}x_j, \quad (i=1,\ 2,\ 3)$$

3、正交条件对变换系数的具体要求

$$\sum_{i=1}^{3} x_i' x_i' = \left(a_{ij}x_j\right)\left(a_{ik}x_k\right)$$

$$= a_{ij}a_{ik}x_j x_k$$

$$= \sum_{j=1}^{3}\sum_{k=1}^{3}\left(\sum_{i=1}^{3} a_{ij}a_{ik}\right)x_j x_k$$

$$\sum_{i=1}^{3} x_i' x_i' = \sum_{j=1}^{3}\sum_{k=1}^{3}\left(\sum_{i=1}^{3} a_{ij}a_{ik}\right)x_j x_k$$

另一方面

$$\sum_{i=1}^{3} x_i x_i = \sum_{j=1}^{3}\sum_{k=1}^{3} \delta_{jk} x_j x_k$$

所以

$$\sum_{i=1}^{3} a_{ij}a_{ik} = \delta_{jk}, \quad (j,k=1,2,3)$$

——称为正交变换条件

简写成： $a_{ij}a_{ik} = \delta_{jk}$

正交条件的矩阵形式：

一般地，两个3×3的矩阵a、b相乘，得c矩阵：

$$\begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix}$$

则等式可写成： $$\sum_{i=1}^{3} b_{ji} a_{ik} = c_{jk}$$

或者 $$b_{ji} a_{ik} = c_{jk}$$

$$\sum_{i=1}^{3} b_{ji} a_{ik} = c_{jk}$$

正交条件：

$$\sum_{i=1}^{3} a_{ij} a_{ik} = \delta_{jk}, \quad (j, k = 1, 2, 3)$$

$$b_{ji} = a_{ij}, \quad \Rightarrow \quad b = \left[b_{ij}\right] = \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{33} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}$$

正交条件：

$$\begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

正交条件： $ba = I$

$$b = \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{23} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}$$

定义转置矩阵：

矩阵 a 的转置矩阵 ã 定义为，

$$\tilde{a}_{ij} = a_{ji},\qquad \tilde{a} = \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{23} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}$$

正交条件：若一个变换矩阵的转置矩阵与变换矩阵本身的乘积为单位矩阵，则这样的变换为正交变换。

$$\tilde{a}\,a = I$$

$$\begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

4、逆变换：

得到其逆变换的形式为

$\tilde{a}$

$\tilde{a}$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix} \begin{bmatrix} x_1' \\ x_2' \\ x_3' \end{bmatrix}$$

或者 $x_j = \sum_k \tilde{a}_{jk} x_k' = \sum_k a_{kj} x_k'$

二. 物理量按空间变换性质的分类

物理量在三维空间转动下的变换性质来划分：

标量、矢量、张量等

1、**标量**

① 物理量在三维空间无取向性；

② 在三维坐标系转动时，物理量保持不变。

③ 例：质量、电荷

2、矢量

① 物理量在三维空间有一定的取向性；

② 在三维空间有三个分量；在空间坐标发生转动时，三个分量按照同一方式变换

③ 例：速度（$\vec{v}$）、力（$\vec{F}$）、电场强度（$\vec{E}$）、磁场强度（$\vec{H}$）

3、二阶张量

1）具有较复杂的空间取向性质（如电四极矩）

2）用并矢表示，共有9个分量；

$$\overleftrightarrow{T}=\begin{pmatrix} T_{11} & T_{12} & T_{13} \\ T_{21} & T_{22} & T_{23} \\ T_{31} & T_{32} & T_{33} \end{pmatrix}\begin{pmatrix} \vec{e}_1\vec{e}_1 & \vec{e}_2\vec{e}_1 & \vec{e}_3\vec{e}_1 \\ \vec{e}_1\vec{e}_2 & \vec{e}_2\vec{e}_2 & \vec{e}_3\vec{e}_2 \\ \vec{e}_1\vec{e}_3 & \vec{e}_2\vec{e}_3 & \vec{e}_3\vec{e}_3 \end{pmatrix}$$

$$\overleftrightarrow{T}=\sum_{i,j}T_{ij}\vec{e}_i\vec{e}_j, \quad (i,j=1,2,3)$$

$$x_j' = a_{jk} x_k$$

$$\overleftrightarrow{T} = T_{ij}\vec{e}_i\vec{e}_j, \quad (i, j = 1, 2, 3)$$

3）在发生某种空间转动时，

$$\overleftrightarrow{T} = T_{ij}'\vec{e}_i'\vec{e}_j', \quad (i, j = 1, 2, 3)$$

$$T_{ij}' = \sum_{k=1}^{3}\sum_{l=1}^{3} a_{ik} a_{jl} T_{kl}$$

$$\overset{\gg}{\mathrm{T}} = \begin{pmatrix} T_{11} & T_{12} & T_{13} \\ T_{21} & T_{22} & T_{23} \\ T_{31} & T_{32} & T_{33} \end{pmatrix} \begin{pmatrix} \vec{e}_1\vec{e}_1 & \vec{e}_2\vec{e}_1 & \vec{e}_3\vec{e}_1 \\ \vec{e}_1\vec{e}_2 & \vec{e}_2\vec{e}_2 & \vec{e}_3\vec{e}_2 \\ \vec{e}_1\vec{e}_3 & \vec{e}_2\vec{e}_3 & \vec{e}_3\vec{e}_3 \end{pmatrix}$$

4）张量的迹：

$$\mathrm{Trace}(T) = \sum_{i=1}^{3} T_{ii}$$

——张量的迹是一个标量

$$T_{ij}' = \sum_{k=1}^{3}\sum_{l=1}^{3} a_{ik}a_{jl}T_{kl}$$

5）二阶对称张量：

① 定义： $T_{ij} = T_{ji}$

② 二阶对称张量在空间转动变换下仍为对称张量

$$T_{ij}' = \sum_{k=1}^{3}\sum_{l=1}^{3} a_{ik}a_{jl}T_{kl} = \sum_{k=1}^{3}\sum_{l=1}^{3} a_{ik}a_{jl}T_{lk}$$

$$= \sum_{k=1}^{3}\sum_{l=1}^{3} a_{il}a_{jk}T_{kl} = \sum_{k=1}^{3}\sum_{l=1}^{3} a_{jk}a_{il}T_{kl} = T_{ji}'$$

6）二阶反对称张量：

① 定义： $T_{ij} = -T_{ji}$

② 二阶反对称张量性质：

- 迹为零
- 在空间转动变换下仍为反对称张量；

三、洛仑兹变换的四维形式

三维空间的转动是满足**距离不变**的线性变换；

$$x_1'^2+x_2'^2+x_3'^2=x_1^2+x_2^2+x_3^2$$

不同惯性参照新之间的空时坐标变换——Lorentz变换满足**间隔不变性**

$$c^2t'^2-x_1'^2-x_2'^2-x_3'^2=c^2t^2-x_1^2-x_2^2-x_3^2$$

或者

$$x_1'^2+x_2'^2+x_3'^2-c^2t'^2=x_1^2+x_2^2+x_3^2-c^2t^2$$

$$x_1'^2+x_2'^2+x_3'^2-c^2t'^2$$
$$=x_1^2+x_2^2+x_3^2-c^2t^2$$

1、定义第四维坐标：

$$x_4=\mathrm{i}ct$$

惯性参照系变换下间隔不变性表示成

$$\sum_{i=1}^{4}x_i'^2=\sum_{i=1}^{4}x_i^2$$

2、Lorentz变换的四维形式

$$\beta = \frac{v}{c}$$

$$x' = \gamma\left(x - \beta ct\right)$$

$$\gamma = 1\Big/\sqrt{1-\beta^2}$$

$$x' = \gamma\left[x_1 + i\beta\left(ict\right)\right]$$

或者

$$x_1' = \gamma\left(x_1 + \mathrm{i}\beta x_4\right)$$

$$t' = \gamma\left(-\frac{\beta}{c}x + t\right),$$

$$\mathrm{i}ct' = \gamma\left(-i\beta x + \mathrm{i}ct\right),$$

$$x_1' = \gamma\left(x_1 + \mathrm{i}\beta x_4\right)$$

$$x_2' = x_2$$

$$x_3' = x_3$$

$$x_4' = \gamma\left(-i\beta x_1 + x_4\right)$$

或者 $x_4' = \gamma\left(-i\beta x_1 + x_4\right),$

四维空间，Lorentz变换的矩阵形式：

$$\begin{bmatrix} x_1' \\ x_2' \\ x_3' \\ x_4' \end{bmatrix} = \begin{bmatrix} \gamma & 0 & 0 & \mathrm{i}\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\mathrm{i}\beta\gamma & 0 & 0 & \gamma \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$$\beta = \frac{v}{c}$$

$$\gamma = 1\Big/\sqrt{1-\beta^2}$$

或写成： $x_\mu' = a_{\mu\nu} x_\nu$

其中

$$x' = \begin{bmatrix} x_1' \\ x_2' \\ x_3' \\ x_4' \end{bmatrix}, \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \quad a = \begin{bmatrix} \gamma & 0 & 0 & \mathrm{i}\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\mathrm{i}\beta\gamma & 0 & 0 & \gamma \end{bmatrix}$$

容易验证Lorentz变换满足正交变换条件：

$$\tilde{a}\, a = I$$

$$\beta = \frac{v}{c}$$

$$\gamma = 1\Big/\sqrt{1-\beta^2}$$

$$a = \begin{bmatrix} \gamma & 0 & 0 & \mathrm{i}\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\mathrm{i}\beta\gamma & 0 & 0 & \gamma \end{bmatrix} \qquad \tilde{a} = \begin{bmatrix} \gamma & 0 & 0 & \text{-i}\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \mathrm{i}\beta\gamma & 0 & 0 & \gamma \end{bmatrix}$$

$$\tilde{a}\,a = \begin{bmatrix} \gamma & 0 & 0 & \mathrm{i}\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \text{-i}\beta\gamma & 0 & 0 & \gamma \end{bmatrix} \bullet \begin{bmatrix} \gamma & 0 & 0 & \text{-i}\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \mathrm{i}\beta\gamma & 0 & 0 & \gamma \end{bmatrix} = \begin{bmatrix} \gamma^2-\beta^2\gamma^2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \gamma^2-\beta^2\gamma^2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = I$$

$$\begin{bmatrix} x_1' \\ x_2' \\ x_3' \\ x_4' \end{bmatrix} = \begin{bmatrix} \gamma & 0 & 0 & \mathrm{i}\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\mathrm{i}\beta\gamma & 0 & 0 & \gamma \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

3、Lorentz变换的逆变换形式

$$x_\mu = \tilde{a}_{\mu\nu} x_\nu' = a_{\nu\mu} x_\nu'$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} \gamma & 0 & 0 & -\mathrm{i}\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \mathrm{i}\beta\gamma & 0 & 0 & \gamma \end{bmatrix} \begin{bmatrix} x_1' \\ x_2' \\ x_3' \\ x_4' \end{bmatrix}$$

四、四维协变量

$$\begin{bmatrix} x_1' \\ x_2' \\ x_3' \\ x_4' \end{bmatrix} = \begin{bmatrix} \gamma & 0 & 0 & \mathrm{i}\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\mathrm{i}\beta\gamma & 0 & 0 & \gamma \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

1、物理量的协变性

① 按照相对论的观点，时间和空间统一在四维空间；

$$x_4 = \mathrm{i}ct$$

② 惯性参照系的空时坐标变换——Lorentz变换相当于四维空间的一种转动；

③ 物理量在Lorentz变换下的不变性称为物理量的协变性

2、按照物理量在四维空间转动下的变换性质，将物理量分类：

1） Lorentz标量

2） 四维矢量

3） 四维张量

1）Lorentz标量

① 在Lorentz变换下，保持不变的物理量；

② 例：**两个事件的间隔，电荷的电量、波的位相**

- 在不同参照系中，测量得到的波矢和角频率不同；
- 由于**波形**是相同的，因此在不同的惯性参照系中的位相是相同的，即位相是Lorentz不变量

$$\phi = \vec{k} \cdot \vec{x} - \omega t = \text{const.}$$

$$a=\begin{bmatrix}\gamma & 0 & 0 & \mathrm{i}\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\mathrm{i}\beta\gamma & 0 & 0 & \gamma\end{bmatrix}$$

2）四维矢量

① 该物理量有四个分量（如四维空时坐标）；

② 在不同的惯性参照系之间，四维矢量的变换关系为Lorentz变换：

$$U'=aU$$

或者 $$U_\mu'=a_{\mu\nu}U_\nu$$

3）四维张量

满足以下变换关系的物理量称为四维张量

$$T_{\mu\nu}' = a_{\mu\lambda} a_{\nu\delta} T_{\lambda\delta}$$

四维张量由16个分量。

五、多普勒效应和光行差

The Doppler effect

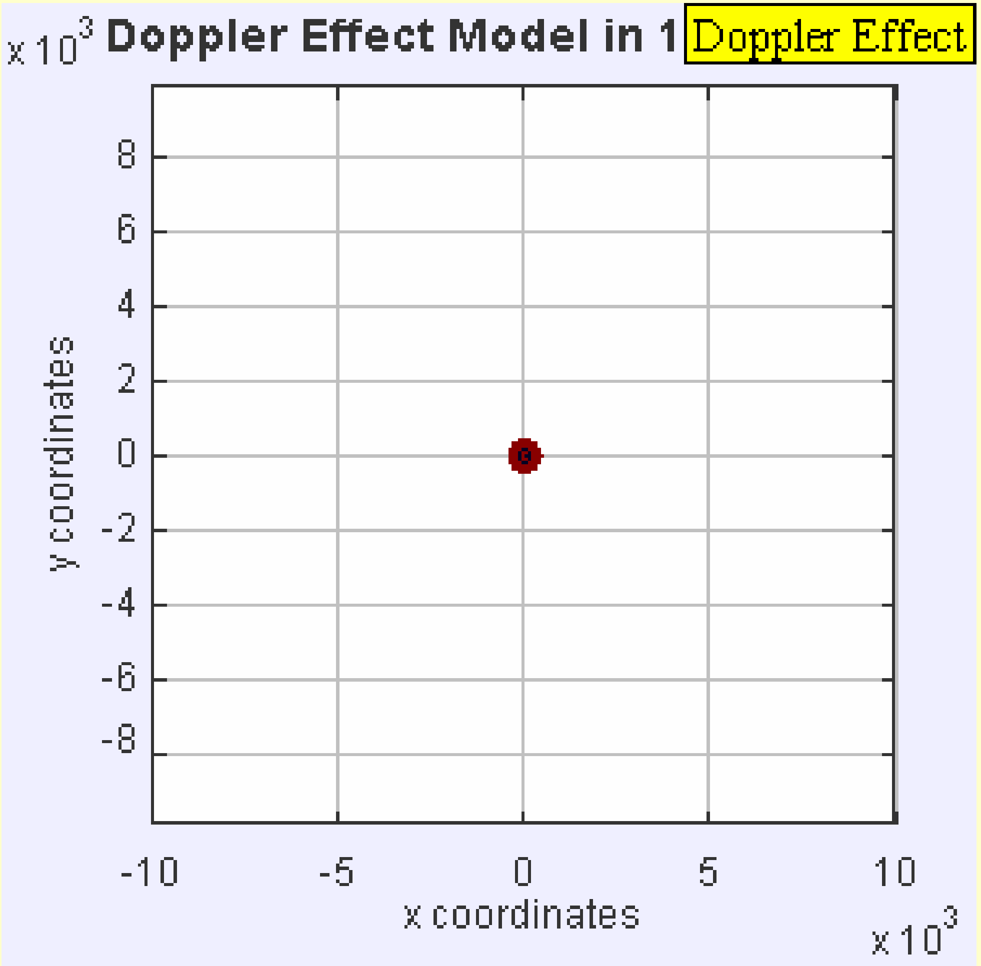
- Doppler effect (or Doppler shift), named after the Austrian physicist Christian Doppler.
- Proposed in 1842 the change in frequency of a wave (or other periodic event) for an observer moving relative to its source.

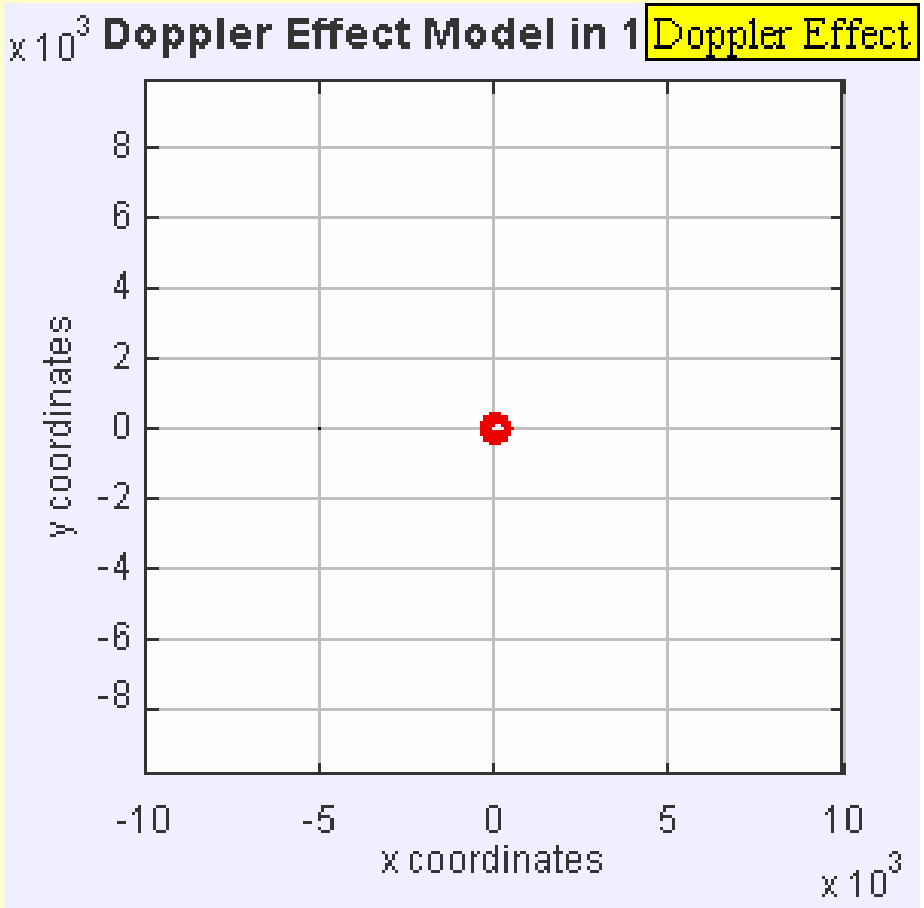
For waves that propagate in a medium

- For waves that propagate in a medium, such as sound waves, the velocity of the observer and of the source are relative to the medium in which the waves are transmitted.
- The total Doppler effect may therefore result from motion of the source, motion of the observer, or motion of the medium.
- Each of these effects is analyzed separately.

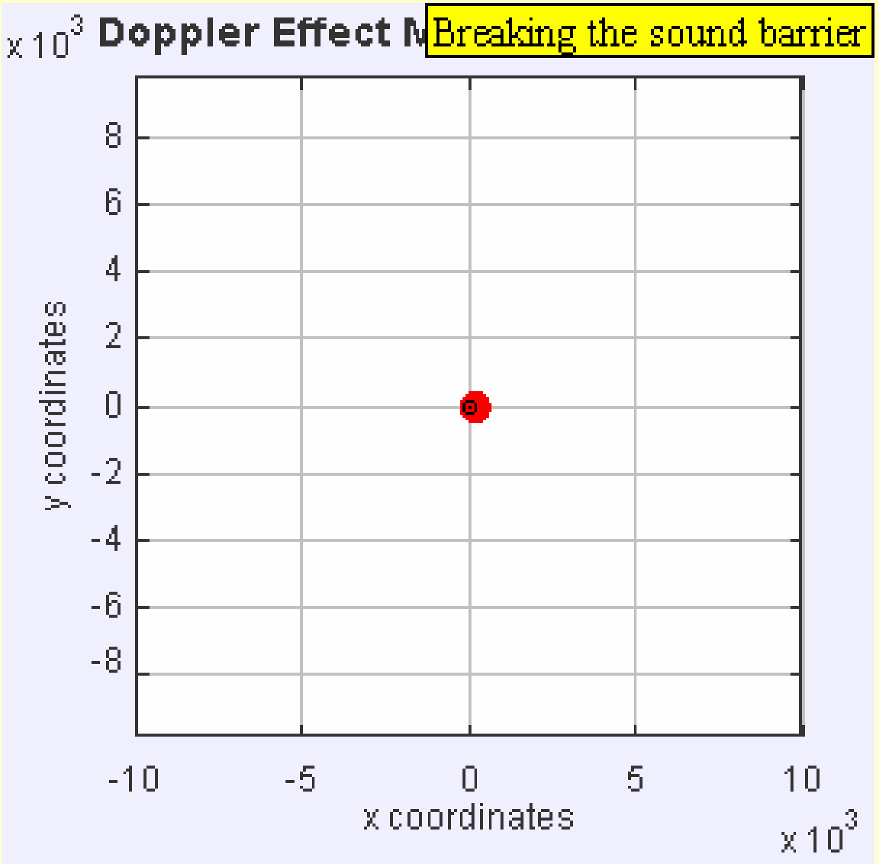
Sound wave-fronts propagate symmetrically away from the source at a constant speed c.

Sound source is moving with a speed $u_s = 0.7\ c$

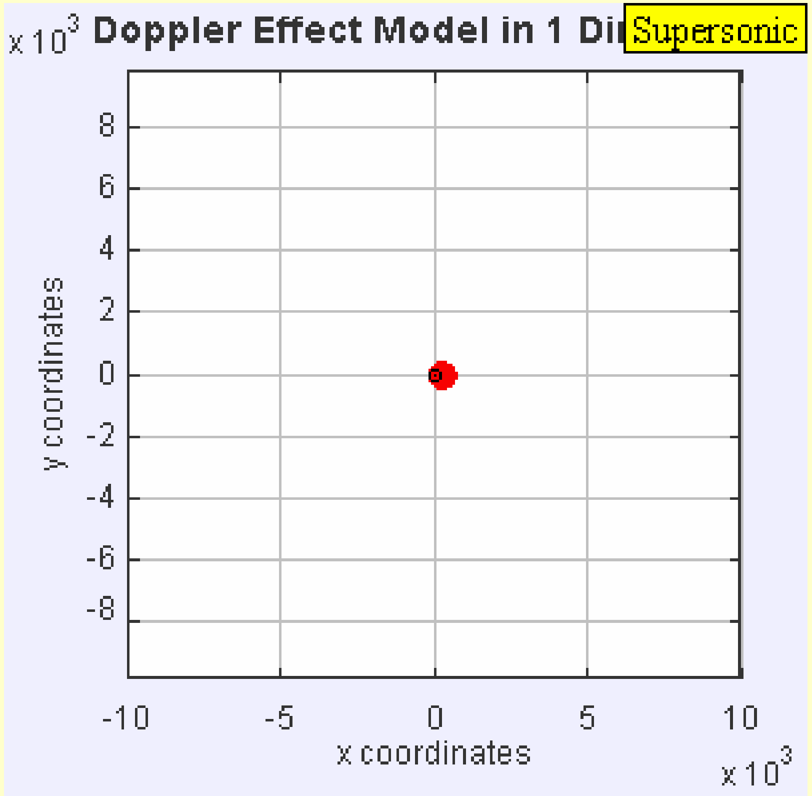




Sound source is moving at the speed of sound in the medium ($u_s = c$, or Mach 1).



Sound source has now broken through the sound speed barrier, and is traveling at 1.4 c (Mach 1.4)



Applications of Doppler effect

- *Astronomy*
- *Temperature measurement*
- *Radar*
- *Medical imaging and blood flow measurement*
- *Satellite communication*
- *Vibration measurement*

For waves which do not require a medium, such as light

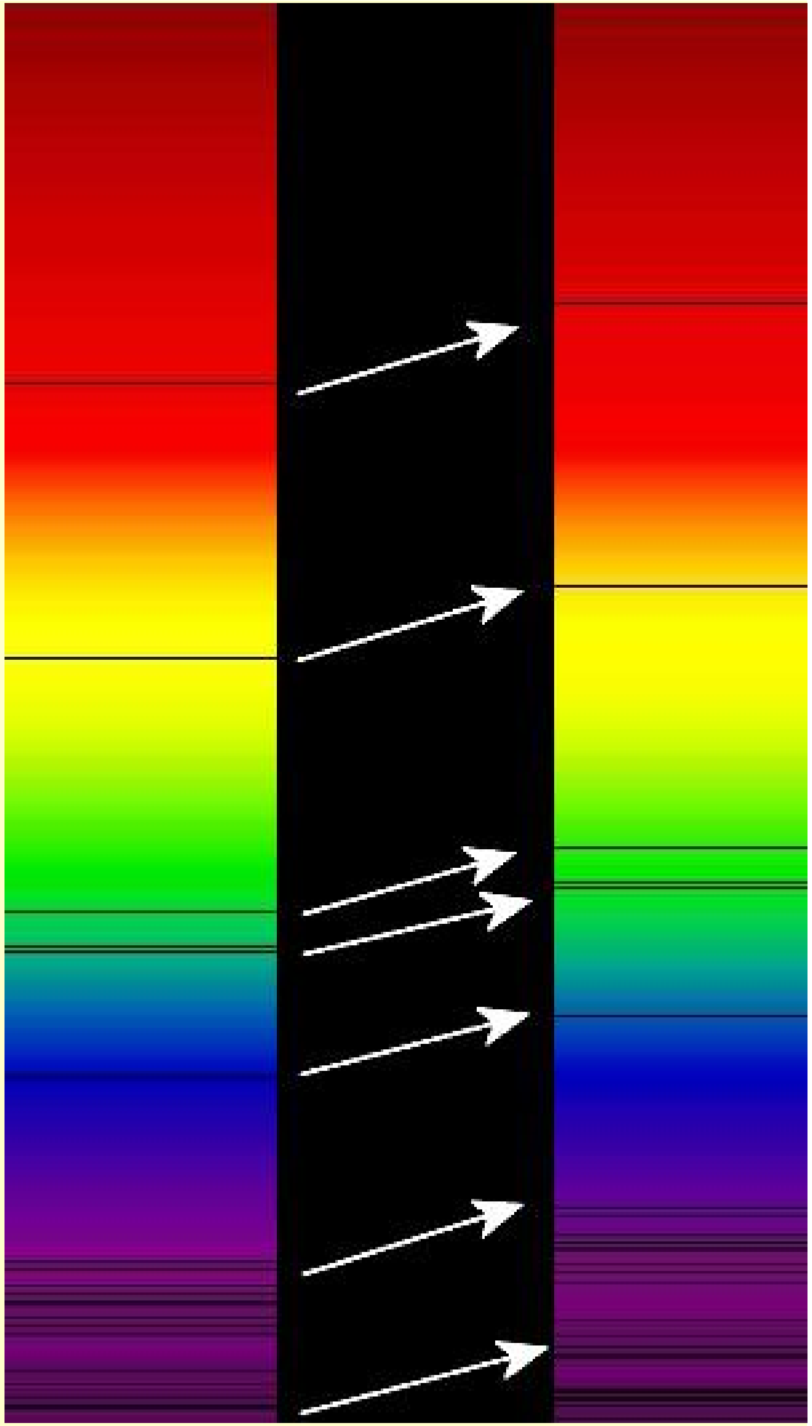
- For waves which do not require a medium, such as light or gravity in general relativity, only the relative difference in velocity between the observer and the source needs to be considered.

Astronomy

- The Doppler effect for EM waves such as light is of great use in astronomy and results in either a so-called redshift or blueshift.
- It has been used to measure the speed at which stars and galaxies are approaching or receding from us, that is, the radial velocity.
- This is used to detect if an apparently single star is, in reality, a close binary and even to measure the rotational speed of stars and galaxies

Absorption lines

- absorption lines are not always at the frequencies that are obtained from the spectrum of a stationary light source.
- Redshift of spectral lines in the optical spectrum of a supercluster of distant galaxies (right), as compared to that of the Sun (left)



1、谱线红移

1）谱线红移现象：

与实验室拍摄的静止光源的（例如电离钙的H、K吸收谱线）特征谱线相比，来自遥远星系的特征谱线的波长向长波长方向移动，这种现象称为**谱线红移**。

2）谱线红移的原因

① 多普勒效应：原子辐射的频率依赖于辐射源与观测者的相对运动；

② 引力红移。

2、四维波矢

1）位相是Lorentz标量

$$\phi = \vec{k} \cdot \vec{x} - \omega t = \text{const.}$$

① 在 Σ' 参照系中光波波矢、角频率（k', ω'）;

② 在 Σ 参照系中光波波矢、角频率(k, ω)。

位相：$\phi = \vec{k} \cdot \vec{x} - \omega t$

$$= k_1 x_1 + k_2 x_2 + k_3 x_3 + \left(\mathrm{i} \frac{\omega}{c} \right) (\mathrm{i} c t)$$

$$x_4 = \mathrm{i}ct$$

2）定义四维波矢：$k_\mu = \left(k_1,\ k_2,\ k_3,\ \mathrm{i}\dfrac{\omega}{c} \right)$

四维波矢的第四分量：

$$k_4 = \mathrm{i}\frac{\omega}{c}$$

位相：$\phi = \vec{k} \cdot \vec{x} - \omega t$ $= k_1 x_1 + k_2 x_2 + k_3 x_3 + \left(\mathrm{i}\dfrac{\omega}{c} \right)(\mathrm{i}ct)$

在Lorentz变换下位相不变的性质可写成

$$\phi = k_\mu x_\mu = \text{const.}$$

$$k_\mu = \left(k_1,\ k_2,\ k_3,\ \mathrm{i}\frac{\omega}{c}\right)$$

3、不同惯性系中四维波矢的变换关系

四维波矢为四维矢量，故变换关系为

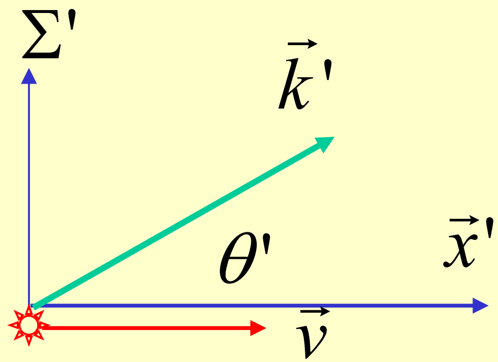
$$k_\mu' = a_{\mu\nu} k_\nu$$

或者

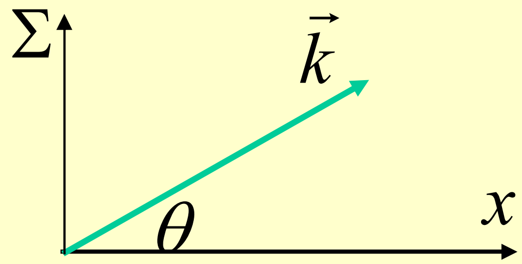
$$\begin{bmatrix} k_1' \\ k_2' \\ k_3' \\ \mathrm{i}\dfrac{\omega'}{c} \end{bmatrix} = \begin{bmatrix} \gamma & 0 & 0 & \mathrm{i}\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\mathrm{i}\beta\gamma & 0 & 0 & \gamma \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ \mathrm{i}\dfrac{\omega}{c} \end{bmatrix}$$

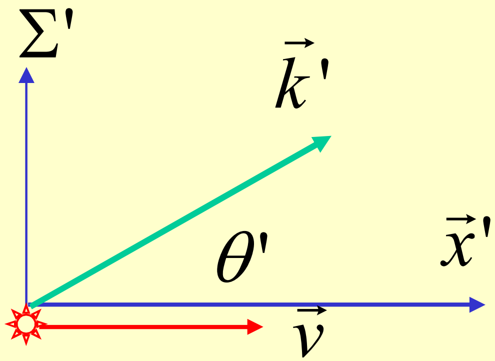
4、两个现象的定量分析

- **多普勒效应**——在不同的参照系中观察到的频率存在差异；



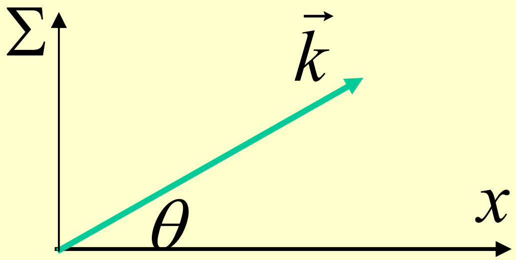
- **光行差**——在光源运动时，光的传播方向亦发生变化；





$$k_x' = \frac{\omega'}{c}\cos\theta',$$

$$k_y' = \frac{\omega'}{c}\sin\theta',$$



$$k_x = \frac{\omega}{c}\cos\theta,$$

$$k_y = \frac{\omega}{c}\sin\theta,$$

$$\begin{bmatrix} k_x' \\ k_y' \\ k_z' \\ \mathrm{i}\dfrac{\omega'}{c} \end{bmatrix} = \begin{bmatrix} \gamma & 0 & 0 & \mathrm{i}\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\mathrm{i}\beta\gamma & 0 & 0 & \gamma \end{bmatrix} \begin{bmatrix} k_x \\ k_y \\ k_z \\ \mathrm{i}\dfrac{\omega}{c} \end{bmatrix}$$

$$k_x' = \frac{\omega'}{c}\cos\theta', \qquad k_x = \frac{\omega}{c}\cos\theta,$$

$$k_y' = \frac{\omega'}{c}\sin\theta', \qquad k_y = \frac{\omega}{c}\sin\theta,$$

根据Lorentz变换：

$$\begin{cases} k_x' = \gamma k_x + \mathrm{i}\beta\gamma k_4, \\ k_y' = k_y, \\ \mathrm{i}\dfrac{\omega'}{c} = -\mathrm{i}\beta\gamma k_x + \gamma\left(\mathrm{i}\dfrac{\omega}{c}\right) \end{cases}$$

$$\begin{cases} k_x' = \gamma k_x + \mathrm{i}\beta\gamma k_4, \\ k_y' = k_y, \\ \mathrm{i}\dfrac{\omega'}{c} = -\mathrm{i}\beta\gamma k_x + \gamma\left(\mathrm{i}\dfrac{\omega}{c}\right) \end{cases}$$

$$\begin{cases} \dfrac{\omega'}{c}\cos\theta' = \gamma\left(\dfrac{\omega}{c}\cos\theta - \dfrac{\omega v}{c^2}\right), \\ \mathrm{i}\dfrac{\omega'}{c} = -\mathrm{i}\gamma\dfrac{\omega v}{c^2}\cos\theta + \mathrm{i}\gamma\dfrac{\omega}{c} \end{cases}$$

求解得： $\omega' = \left(-\dfrac{v}{c}\cos\theta + 1\right)\gamma\omega$

$$\mathrm{tg}\,\theta' = \frac{\sin\theta}{\gamma\left(\cos\theta - \dfrac{v}{c}\right)}$$

$$\omega' = \gamma\left(-\frac{v}{c}\cos\theta + 1\right)\omega$$

$$\mathrm{tg}\,\theta' = \frac{\sin\theta}{\gamma\left(\cos\theta - \frac{v}{c}\right)}$$

① 第一个关系式表示：在不同的参照系中观察到的频率存在差异——多普勒效应；

② 第二个关系式表示：在光源运动时，光的传播方向亦发生变化——光行差。

$$\omega' = \gamma\left(-\frac{v}{c}\cos\theta + 1\right)\omega$$

多普勒效应：

① Σ’为光源静止的参照系，ω’ ＝ ω_0为静止光源的辐射频率；

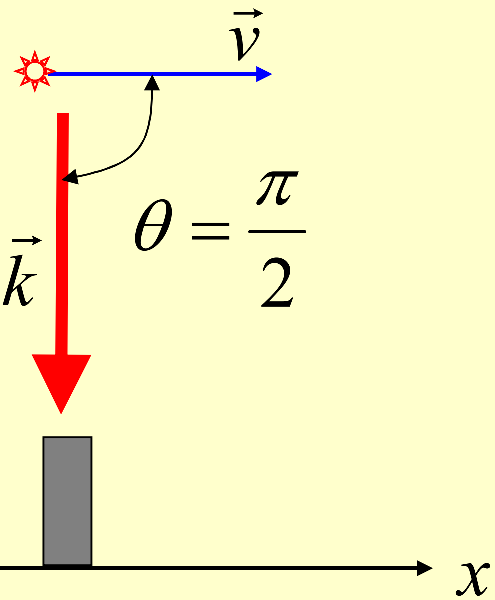
② 运动光源辐射的角频率为

$$\omega = \frac{\omega_0}{\gamma\left(1 - \frac{v}{c}\cos\theta\right)},$$

$$\omega = \frac{\omega_0}{\gamma\left(1 - \frac{v}{c}\cos\theta\right)},$$

（a）在垂直于光源的运动方向观测辐射

$$\omega = \omega_0\sqrt{1 - \frac{v^2}{c^2}} < \omega_0$$

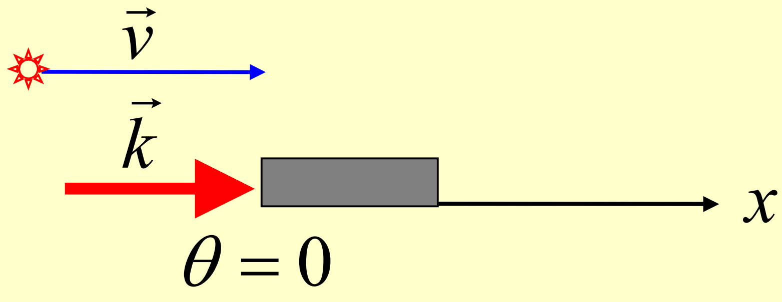


此时，观测到的辐射频率小于静止光源的辐射频率——**横向多普勒效应**。

$$\omega = \frac{\omega_0}{\gamma\left(1 - \frac{v}{c}\cos\theta\right)},$$

（b）光源向着观测者方向运动

$$\theta = 0$$



$$\omega = \frac{\sqrt{1 + v/c}}{\sqrt{1 - v/c}}\omega_0 > \omega_0$$

$$\lambda = \frac{\sqrt{1 - v/c}}{\sqrt{1 + v/c}}\lambda_0 < \lambda_0$$

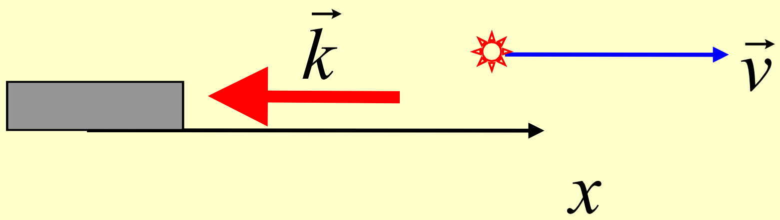
此时，观测到的辐射频率大于静止光源的辐射频率；辐射的波长减小（**波长发生蓝移**）。

$$\omega = \frac{\omega_0}{\gamma\left(1-\frac{v}{c}\cos\theta\right)},$$

（c）光源远离观测者而去 $(\theta = \pi)$

$$\omega = \frac{\sqrt{1-v/c}}{\sqrt{1+v/c}}\omega_0 < \omega_0$$

$$\lambda = \frac{\sqrt{1+v/c}}{\sqrt{1-v/c}}\lambda_0 > \lambda_0$$



- 观测到的辐射频率小于静止光源的辐射频率；而辐射的波长增大（**波长发生红移**）；
- 根据来自星系辐射的红移量，可以估算出星系远离地球的速度——**宇宙膨胀**。

Positive radial velocity means the star is receding from the Sun, negative that it is approaching.

- Since blue light has a higher frequency than red light, the spectral lines of an approaching astronomical light source exhibit a blueshift and those of a receding astronomical light source exhibit a redshift.
- Among the nearby stars, the largest radial velocities with respect to the Sun are +308 km/s (BD-15° 4041, also known as LHS 52, 81.7 light-years away)
- and -260 km/s (Woolley 9722, also known as Wolf 1106 and LHS 64, 78.2 light-years away).

Mass–energy equivalence

- Mass–energy equivalence is a consequence of special relativity.
- The energy and momentum, which are separate in Newtonian mechanics, form a four-vector in relativity, and this relates the time component (the energy) to the space components (the momentum) in a nontrivial way.
- For an object at rest, the energy–momentum four-vector is (E, 0, 0, 0): it has a time component which is the energy, and three space components which are zero.
- By changing frames with a Lorentz transformation in the x direction with a small value of the velocity v, the energy momentum four-vector becomes (E, Ev/c2, 0, 0). The momentum is equal to the energy multiplied by the velocity divided by c2.
- As such, the Newtonian mass of an object, which is the ratio of the momentum to the velocity for slow velocities, is equal to E/c2.

考虑补充关于利用光的Doppler效应来冷却原子的知识

- http://en.wikipedia.org/wiki/Doppler_effect