第七章
ZT

7.1 ZT定义

1、引言

- LTI模拟系统
  - 常系数微分方程
  - 时域法或LT求解
  - H(s)零极点分析系统的时域及频域响应
- LTI离散系统
  - 常系数差分方程
  - 时域迭代法或ZT求解
  - H(z)零极点分析系统
- LT及ZT都是FT的推广

2、定义

– 抽样信号的LT

$f_s = 1/T$对$x(t)$进行抽样

$$x_s(t) = x(t)\delta_T(t) = x(t)\delta(t) + x(t)\delta(t-T) + x(t)\delta(t-2T) + \ldots$$

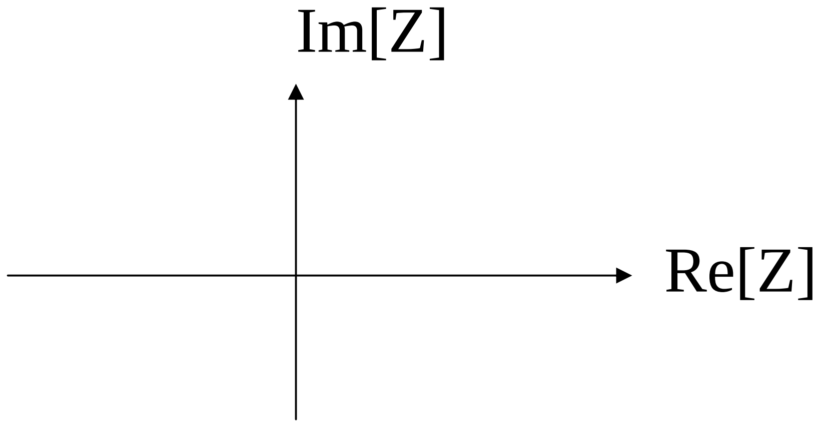
$$X_s(s) = \int_0^{\infty} x_s(t)e^{-st}dt = x(0) + x(T)e^{-sT} + x(2T)e^{-s2T} + \ldots$$

$$= \sum_{n=0}^{\infty} x(nT)e^{-nTs}$$ (抽样信号的LT是S域的级数)

令$Z = e^{sT}, s = \frac{1}{T}\ln Z$

$$\therefore X(z) = \sum_{n=0}^{\infty} x(n)z^{-n}$$

– Z平面



– 双边ZT

$$X(z)=\sum_{n=-\infty}^{\infty}x(n)z^{-n}$$

– 单边ZT

$$X(z)=\sum_{n=0}^{\infty}x(n)z^{-n}$$

7.2 ZT的收敛域ROC

1、ZT的ROC

– 对于任一序列，使ZT收敛的所有Z的集合

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n} \longleftarrow \sum_{n=-\infty}^{\infty} \left| x(n) z^{-n} \right| < \infty$$

$$z = re^{j\theta} \longrightarrow \sum_{n=-\infty}^{\infty} \left| x(n) r^{-n} \right| < \infty$$

FT存在条件 $\longrightarrow \int_{-\infty}^{\infty} | x(t) | dt < \infty$

LT存在条件 $\longrightarrow \int_{-\infty}^{\infty} | x(t) | e^{-\sigma t} dt < \infty$

2、判定正项级数收敛 $\sum_{n=-\infty}^{\infty}|a_n|$

– 比例判定法(后项与前项比值的极限)

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| \begin{cases} <1 & \text{收敛} \\ >1 & \text{发散} \\ =1 & \text{可能收敛，可能发散} \end{cases}$$

– 根值判定法(n次根的限)

$$\lim_{n\to\infty}\sqrt[n]{|a_n|} \begin{cases} <1 & \text{收敛} \\ >1 & \text{发散} \\ =1 & \text{可能收敛，可能发散} \end{cases}$$

例：序列$x(n)=a^n u(n)$(实指数序列），求ZT

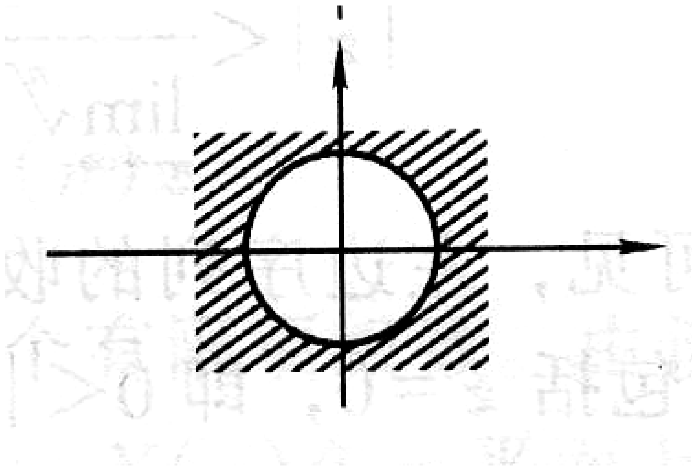
$$Z[x(n)]=\sum_{n=-\infty}^{\infty}a^n u(n)z^{-n}=\sum_{n=0}^{\infty}a^n z^{-n}=\sum_{n=0}^{\infty}(az^{-1})^n$$

比值判定法：

$|az^{-1}|<1$,其和收敛，且

$$\mathrm{Z}[x(n)]=\frac{1}{1-az^{-1}}=\frac{z}{z-a}$$

$ROC:|z|>|a|$



例：序列$x(n)=-b^n u(-n-1)$(实指数序列），求ZT

$$Z[x(n)]=\sum_{n=-\infty}^{\infty}-b^n u(-n-1)z^{-n}=\sum_{n=-\infty}^{-1}-b^n z^{-n}$$

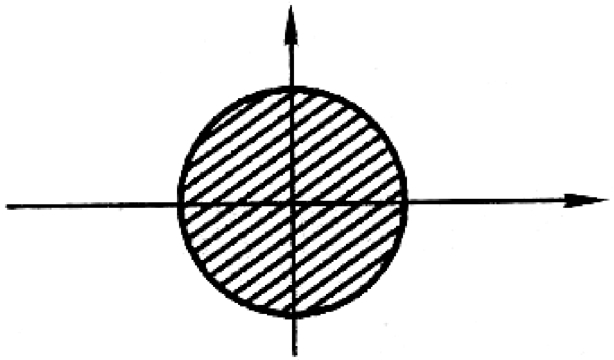
$$=\sum_{m=1}^{\infty}-b^{-m}z^{m}=\sum_{m=0}^{\infty}-b^{-m}z^{m}-(-1)=1-\sum_{n=0}^{\infty}(b^{-1}z)^n$$

比值判定法：

$|b^{-1}z|<1$,其和收敛，且

$$Z[x(n)]=1-\frac{1}{1-b^{-1}z}=\frac{z}{z-b}$$

$ROC:|z|<|b|$



$X(z)=\dfrac{z}{z-a}$, $z=0$零点，$z=a$极点

极点总在ROC边缘，不包含极点

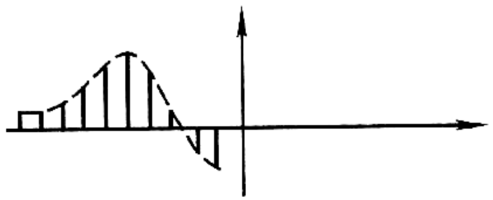
3、典型序列的ROC

– 有限长序列

$$n_1 \le n \le n_2$$

$$X(z) = \sum_{n_1}^{n_2} x(n) z^{-n}$$

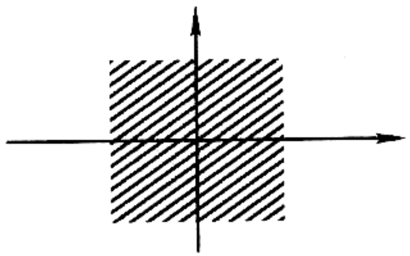
$$(1) n_1 < n_2 < 0$$



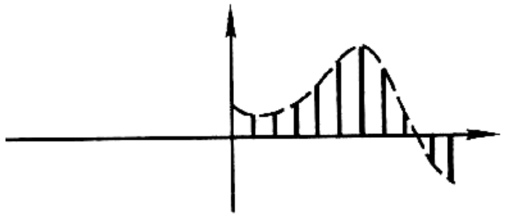
$$X(z) = \sum_{n=|n_2|}^{|n_1|} x(-n) z^{n}$$

除$z = \infty$外，都收敛

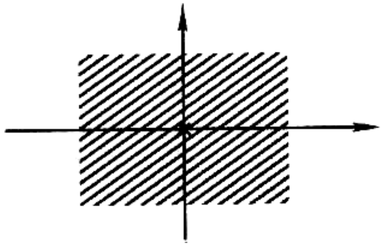
ROC为$:0 \le |z| < \infty$



(2) $0 \le n_1 < n_2$



$$X(z)=\sum_{n=n_1}^{n_2} x(n)z^{-n}$$



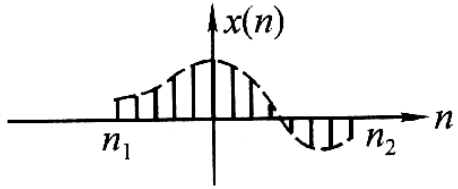
除$z=0$外，都收敛

ROC为: $0<|z|\le\infty$

(3) $n_1<0, n_2>0$

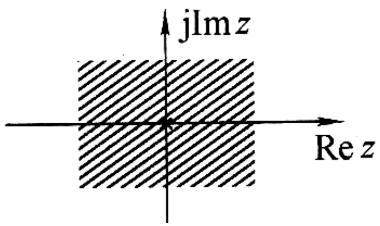
$$X(z)=\sum_{n=n_1}^{n_2} x(n)z^{-n}=\sum_{n=n_1}^{-1} x(n)z^{-n}+\sum_{n=0}^{n_2} x(n)z^{-n}$$

$$=\sum_{n=|n_1|}^{1} x(-n)z^{n}+\sum_{n=0}^{n_2} x(n)z^{-n}$$



除$z=0$及∞外，都收敛

ROC为: $0<|z|<\infty$

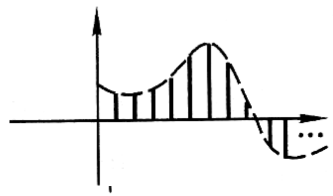


– 右边序列

$n < n_1$时$x(n) = 0$的序列（有始无终）

$$X(z) = \sum_{n_1}^{\infty} x(n) z^{-n}$$

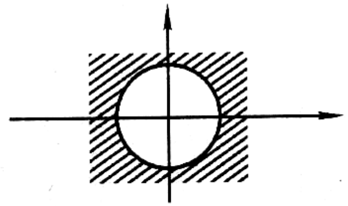
$n_1 \geq 0, n = 0$时右边序列为因果序列



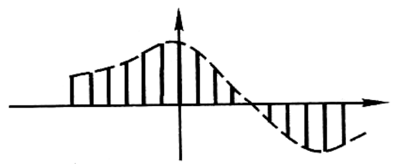
对于$|z| > |z_1|$,有

$$\sum_{n_1}^{\infty} |x(n) z^{-n}| < \sum_{n_1}^{\infty} |x(n) z_1^{-n}| < \infty$$

$\therefore ROC: |z| > |z_1|$



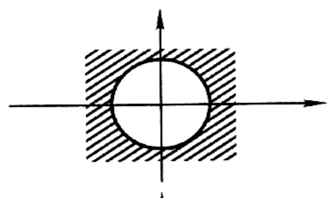
$n_1 < 0$, 对于$|z| > |z_1|$,有



$$\sum_{n_1}^{\infty} |x(n) z^{-n}| = \sum_{n_1}^{-1} |x(n) z^{-n}| + \sum_{0}^{\infty} |x(n) z^{-n}|$$

$Z \neq \infty$　　　　$|z| > |z_1|$

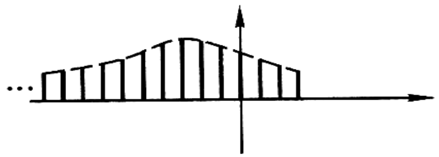
$\therefore ROC: |z_1| < |z| < \infty$



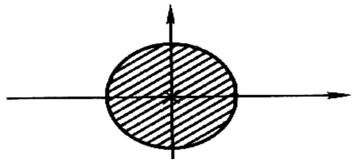
– 左边序列

$n > n_2$时$x(n) = 0$的序列（无始有终）

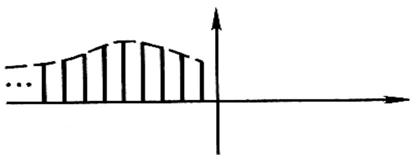
$$X(z) = \sum_{-\infty}^{n_2} x(n) z^{-n}$$



令$m = -n$



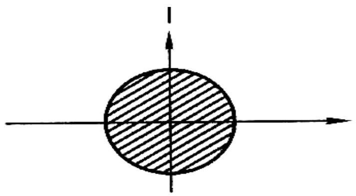
$$X(z) = \sum_{-n_2}^{\infty} x(-m) z^{m}$$



则当$|z| < |z_2|$, ZT收敛

$n_2 > 0, 0 < |z| < |z_2|$

$n_2 \le 0, 0 \le |z| < |z_2|$



– 双边序列

双边序列是指n从$-\infty$到$+\infty$的序列（无始无终）

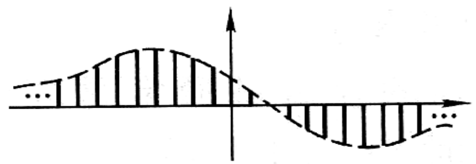
$$X(z)=\sum_{-\infty}^{\infty}x(n)z^{-n}=\sum_{-\infty}^{-1}x(n)z^{-n}+\sum_{0}^{\infty}x(n)z^{-n}$$

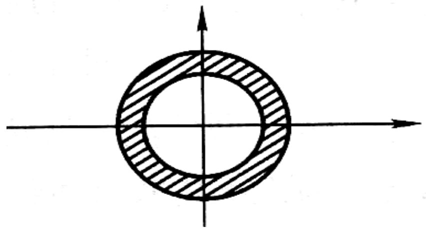
左边序列　　右边序列

$0\le|z|<|z_2|$　　$|z_1|<|z|\le\infty$

$|z_1|<|z_2|$, ROC为$|z_1|<|z|<|z_2|$

$|z_1|>|z_2|$，不存在ROC





例：$x(n)=c^{|n|}, c$为实数，讨论ROC

$$X(z)=\sum_{-\infty}^{\infty}x(n)z^{-n}=\sum_{-\infty}^{-1}c^{-n}z^{-n}+\sum_{0}^{\infty}c^{n}z^{-n}$$

$$=\sum_{1}^{\infty}c^{n}z^{n}+\sum_{0}^{\infty}c^{n}z^{-n}$$

$$=\sum_{0}^{\infty}c^{n}z^{n}-1+\sum_{0}^{\infty}c^{n}z^{-n}$$

$$X_1(z) \qquad X_2(z)$$

$|cz|<1$时，$X_1(z)=\dfrac{1}{1-cz}-1=\dfrac{cz}{1-cz}$

$|cz^{-1}|<1$时，$X2(z)=\dfrac{1}{1-cz^{-1}}=\dfrac{z}{z-c}$

$$\therefore |c|<|z|<|\frac{1}{c}|, X(z)=X_1(z)+X_2(z)$$

7.3　典型序列的单边ZT

1、单位样值序列

$$\delta(n)=\begin{cases}1 & \mathrm{n}=0\\0 & \mathrm{n}\neq 0\end{cases}$$

$$Z[\delta(n)]=\sum_{n=0}^{\infty}\delta(n)z^{-n}=z^{0}=1$$

ROC?

2、单位阶跃序列

$$u(n)=\begin{cases}1 & n\geq 0\\ 0 & n<0\end{cases}$$

$$X(z)=\sum_{n=0}^{\infty}z^{-n}$$

$$|z|>1, X(z)=\frac{1}{1-z^{-1}}=\frac{z}{z-1}$$

3、指数序列

$$x(n) = a^n u(n)$$

$$X(z) = \frac{z}{z-a} \quad (|z| > |a|)$$

$$x(n) = e^{-an} u(n)$$

$$X(z) = \frac{z}{z-e^{-a}} \quad (|z| > |e^{-a}|)$$

7.4　逆Z变换（IZT）

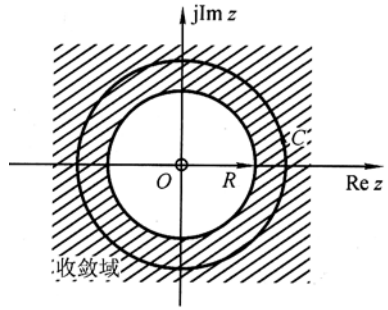
- CTS—常系数微分方程—LT求解—代数方程—S域解—ILT—时域解
- DTS—常系数差分方程—ZT求解—代数方程—Z域解—IZT—时域解
- IZT的三种方法
  - 围线积分法（留数法）
  - 幂级数展开法（长除法）
  - 部分分式展开法

1、围线积分法－留数法

– 柯西定理

回线c所围闭合单通区域上的解析函数$f(z)$沿c的回线积分$\oint f(z)dz=0$

如$f(z)$存在有限个奇点（孤子奇点）



例如$\dfrac{1}{z-a}$的孤立奇点为a,则

$$\oint_c (z-a)^n dz=\begin{cases}2\pi j & n=-1\\ 0 & n\neq -1\text{或}n=-1,\text{但}c\text{不包围}a\end{cases}$$

如果$a=0$

$$\oint_c z^n dz=\begin{cases}2\pi j & n=-1\\ 0 & n\neq -1\end{cases}\text{或}\oint_c z^{k-1}dz=\begin{cases}2\pi j & k=0\\ 0 & k\neq 0\end{cases}$$

$$X(z)=\sum_{n=-\infty}^{\infty}x(n)z^{-n}$$

$$\frac{1}{2\pi j}\oint_c X(z)z^{k-1}dz=\frac{1}{2\pi j}\oint_c \sum_{n=-\infty}^{\infty}x(n)z^{-n}z^{k-1}dz$$

$$=\frac{1}{2\pi j}\oint_c \sum_{n=-\infty}^{\infty}x(n)z^{-n+k-1}dz=\sum_{n=-\infty}^{\infty}x(n)\frac{1}{2\pi j}\oint_c z^{-n+k-1}dz$$

只有当$n=k$时，上述积分才为 1

$$\therefore x(k)=\frac{1}{2\pi j}\oint_c X(z)z^{k-1}dz$$

$$x(n)=\frac{1}{2\pi j}\oint_c X(z)z^{n-1}dz$$

而$\oint_c X(z)z^{n-1}dz=2\pi j\sum[X(z)z^{n-1}$在围线内的极点的留数$]$

$\therefore x(n)=\sum[X(z)z^{n-1}$在围线内的极点的留数$]$

极点留数的求法：

若z_m为$f(z)$的单极点

$$\operatorname{Res}[X(z)z^{n-1}]_{z=z_m}=(z-z_m)X(z)z^{n-1}|_{z=z_m}$$

若z_m为$f(z)$的s阶极点

$$\operatorname{Res}[X(z)z^{n-1}]_{z=z_m}=\frac{1}{(s-1)!}\cdot\frac{d^{s-1}}{dz^{s-1}}[(z-z_m)^s X(z)z^{n-1}]|_{z=z_m}$$

例：$X(z)=\dfrac{4}{4+3z^{-1}}, |z|>\dfrac{3}{4}$，用留数法求$x(n)$

$(1) n \geq 0$时, $X(z)z^{n-1}=\dfrac{z^n}{z+\frac{3}{4}}$, 极点$z=-\dfrac{3}{4}$

$x(n)=[(z+\frac{3}{4})\dfrac{z^n}{z+\frac{3}{4}}]_{z=-\frac{3}{4}}=(-\frac{3}{4})^n \therefore x(n)=(-\frac{3}{4})^n$

$(2) n<0$时, $X(z)z^{n-1}=\dfrac{z^n}{z+\frac{3}{4}}$, 两个极点$\begin{cases} z=-\frac{3}{4}, (\text{一阶极点}) \\ z=0, (n\text{阶极点}) \end{cases}$

$x(n)=x_1(n)+x_2(n)$

$x_1(n)=(-\frac{3}{4})^n, x_2(n)=\dfrac{1}{(m-1)!}\dfrac{d^{m-1}}{dz^{m-1}}[\dfrac{1}{z+\frac{3}{4}}]|_{z=0}=-(-\frac{3}{4})^{-m}$

$\therefore x(n)=x_1(n)+x_2(n)=0$

$\therefore x(n)=(-\frac{3}{4})^n u(n)$

2、幂级数展开法（长除法）

$$X(z)=\sum_{n=-\infty}^{\infty}x(n)z^{-n}$$

- 将X(z)写成Z的幂级数形式，各相应项的系数构成x(n)
- X(z)为有理函数，可用长除法得到幂级数
  - X(z)的ROC为$|z|>|z_1|$，则x(n)是右边序列，X(z)应按z的降幂排列
  - X(z)的ROC为$|z|<|z_2|$，则 x(n)是左边序列，X(z)应按z的升幂排列

例：求$X(z)=\dfrac{z}{z+a}$的IZT，$|z|>a$

ROC在圆外，序列为因果序列，x(n)按降幂排列

$x(n)=(-a)^n u(n)$

例：求$X(z)=\dfrac{-b}{z-b}$的IZT，$|z|<b$

ROC在圆内，序列为左边序列，x(n)按升幂排列

$x(n)=(b)^n u(-n)$

例：求$X(z)=\dfrac{z}{(z-1)^2}$的IZT，$|z|>1$

$$X(z)=\frac{z}{z^2-2z+1}$$

$$
\begin{array}{r}
z^{-1}+2z^{-2}+3z^{-3}+\cdots \\
z^2-2z+1\,\overline{)\,z\qquad\qquad\qquad\qquad} \\
\underline{z-2+z^{-1}\qquad\qquad} \\
2-z^{-1}\qquad\qquad \\
\underline{2-4z^{-1}+2z^{-2}\qquad} \\
3z^{-1}-2z^{-2}\qquad \\
\underline{3z^{-1}-6z^{-2}+3z^{-3}} \\
4z^{-2}-3z^{-3} \\
\cdots
\end{array}
$$

$$X(z)=z^{-1}+2z^{-2}+3z^{-3}+\cdots$$
$$=\sum_{n=0}^{\infty}nz^{-n}$$

$$x(n)=nu(n)$$

3、部分分式展开法

- PFE方法将X(z)展开多个部分分式之和

$$1 \rightarrow \delta(n)$$

$$\frac{z}{z-a} \rightarrow a^n u(n)$$

$$\frac{z}{z-e^{-a}} \rightarrow e^{-an} u(n)$$

- $X(z) = \frac{N(z)}{D(z)}$ 若只有k个一阶极点

$$X(z) = A_0 + \sum_{m=1}^{k} \frac{A_m z}{z - z_m}, A_m = (z - z_m)\frac{X(z)}{z}\Big|_{z=z_m}$$

$$|z > z_1|: x(n) = A_0\delta(n) + \sum_{m=1}^{k} A_m (z_m)^n u(n)$$

$$|z < z_2|: x(n) = A_0\delta(n) - \sum_{m=1}^{k} A_m (z_m)^n u(-n-1)$$

- 若有高阶极点，$z=z_i$为s阶极点

$$X(z)=A_0+\sum_{m=1}^{M}\frac{A_m z}{z-z_m}+\sum_{j=1}^{s}\frac{B_j z}{(z-z_j)^j}$$

$$B_j=\frac{1}{(s-j)!}\left[\frac{d^{s-j}}{dz^{s-j}}(z-z_i)^s\frac{X(z)}{z}\right]\Big|_{z=z_i}$$

$$j=1,2,\dots s$$

例：$X(z)=\dfrac{5z}{z^2+z-6},(2<|z|<3)$，求$x(n)$

$$\frac{X(z)}{z}=\frac{5}{z^2+z-6}=\frac{5}{(z-2)(z+3)}=\frac{A_1}{z-2}+\frac{A_2}{z+3}$$

$$A_1=\operatorname{Res}[\frac{X(z)}{z}]_{z=2}=(z-2)\frac{5}{(z-2)(z+3)}|_{z=2}=1$$

$$A_2=\operatorname{Res}[\frac{X(z)}{z}]_{z=3}=(z+3)\frac{5}{(z-2)(z+3)}|_{z=-3}=-1$$

$$\therefore X(z)=\frac{z}{z-2}-\frac{z}{z+3}$$

$$\because |z|>2,\therefore x_1(n)=z^{-1}[\frac{z}{z-2}]=2^n u(n)$$

$$\because |z|<3,\therefore x_2(n)=z^{-1}[\frac{-z}{z+3}]=(-3)^n u(-n-1)$$

$$\therefore x(n)=x_1(n)+x_2(n)=2^n u(n)+(-3)^n u(-n-1)$$

例： $X(z)=\dfrac{2z^2}{(z+1)(z+2)^2},(|z|>2)$， 求$x(n)$

$$\frac{X(z)}{z}=\frac{2z}{(z+1)(z+2)^2}=\frac{A_1}{z+1}+\frac{B_1}{z+2}+\frac{B_2}{(z+2)^2}$$

$$A_1=\mathrm{Re}\,s[\frac{X(z)}{z}]_{z=-1}=(z+1)\frac{2z}{(z+1)(z+2)^2}|_{z=-1}=-2$$

$$B_1=\frac{d}{dz}[(z+2)^2\frac{X(z)}{z}]|_{z=-2}=\frac{d}{dz}[\frac{2z}{z+1}]|_{z=-2}=\frac{2}{(z+1)^2}|_{z=-2}=2$$

$$B_2=(z+2)^2\frac{X(z)}{z}|_{z=-2}=\frac{2z}{z+1}|_{z=-2}=4$$

$$\therefore X(z)=\frac{-2z}{z+1}+\frac{2z}{z+2}+\frac{4z}{(z+2)^2}$$

$$\because |z|>2,\therefore x(n)=[-2\times(-1)^n+2\times(-2)^n-2\times n(-2)^n]u(n)$$

$$=-2[(-1)^n+(n-1)(-2)^n]u(n)$$

7.5 ZT的基本性质

1、线性特性

$$X(z)=Z[x(n)],(R_{x1}<|z|<R_{x2})$$

$$Y(z)=Z[y(n)],(R_{y1}<|z|<R_{y2})$$

$$Z[ax(n)+by(n)]=aX(z)+bY(z),(R_1<|z|<R_2)$$

$$R_1=\max(R_{x1},R_{y1}),R_2=\min(R_{x2},R_{y2})$$

线性组合可使极点发生变化，可使极点抵消

$$x(n)=a^n u(n)\rightarrow X(z)=\frac{z}{z-a}(|z|>|a|),\quad y(n)=a^n u(n-1)\rightarrow X(z)=\frac{a}{z-a}(|z|>|a|)$$

$$Z[x(n)-y(n)]=1(\text{全z平面})$$

$$Z[e^{an}u(n)] = \frac{z}{z-e^{a}} \quad (|z| > |e^{a}|)$$

$$a = j\omega_0$$

$$Z[e^{j\omega_0 n}u(n)] = \frac{z}{z-e^{j\omega_0}} \quad (|z| > 1)$$

$$Z[e^{-j\omega_0 n}u(n)] = \frac{z}{z-e^{-j\omega_0}} \quad (|z| > 1)$$

$$Z[\cos\omega_0 n u(n)] = \frac{1}{2}\left[\frac{z}{z-e^{j\omega_0}} + \frac{z}{z-e^{-j\omega_0}}\right]$$

$$= \frac{z(z-\cos\omega_0)}{z^2 - 2z\cos\omega_0 + 1}$$

$$Z[\sin\omega_0 n u(n)] = \frac{z\sin\omega_0}{z^2 - 2z\cos\omega_0 + 1}$$

2、移序特性

– 双边ZT

$$Z[x(n)] = X(z) \qquad (|z_1| < |z| < |z_2|)$$

$$Z[x(n \pm n_0)] = Z^{\pm n_0} X(z) \qquad (|z_1| < |z| < |z_2|)$$

$$Z[x(n+n_0)] = \sum_{n=-\infty}^{\infty} x(n+n_0) z^{-n}$$

$$m = n + n_0$$

$$Z[x(n+n_0)] = \sum_{m=-\infty}^{\infty} x(m) z^{-m+n_0} = z^{n_0} X(z)$$

- ZT中引入新的极点，则ROC会发生变化

$$Z[\delta(n)] = 1$$ 全平面

$$Z[\delta(n-1)] = z^{-1}, (|z| > 0)$$

$$Z[\delta(n+2)] = z^{2}, (|z| < \infty),$$ 多了$z = \infty$极点

– 单边ZT

$$Z[x(n)u(n)] = X(z)$$

$$Z[x(n-n_0)u(n)] = z^{-n_0}[X(z) + \sum_{m=-n_0}^{-1} x(m)z^{-m}]$$

$$Z[x(n+n_0)u(n)] = z^{n_0}[X(z) - \sum_{m=0}^{n_0-1} x(m)z^{-m}]$$

$$m = n - n_0$$

$$Z[x(n-n_0)u(n)] = \sum_{m=-n_0}^{\infty} x(m)z^{-m-n_0}$$

$$= z^{-n_0}[\sum_{m=0}^{\infty} x(m)z^{-m} + \sum_{m=-n_0}^{-1} x(m)z^{-m}]$$

$$= z^{-n_0}[X(z) + \sum_{m=n_0}^{-1} x(m)z^{-m}]$$

$$Z[y(n-1)u(n)] = z^{-1}Y(z) + y(-1)$$

$$Z[y(n-2)u(n)] = z^{-2}Y(z) + z^{-1}y(-1) + y(-2)$$

$$Z[y(n+1)u(n)] = zY(z) - zy(0)$$

$$Z[y(n+2)u(n)] = z^{2}Y(z) - z^{2}y(0) - zy(1)$$

3、z域微分特性（序列线性加权）

$$Z[x(n)] = X(z)$$

$$Z[nx(n)] = -z\frac{d}{dz}X(z)$$

$$-z\frac{d}{dz}X(z) = -z\frac{d}{dz}[\sum_{n=-\infty}^{\infty}x(n)z^{-n}] = -z\sum_{n=-\infty}^{\infty}x(n)\frac{d}{dz}z^{-n}$$

$$= -z\sum_{n=-\infty}^{\infty}x(n)(-n)z^{-n-1} = \sum_{n=-\infty}^{\infty}nx(n)z^{-n} = Z[nx(n)]$$

求$:Z[na^n u(n)]$

$$Z[a^n u(n)] = \frac{z}{z-a}$$

$$Z[na^n u(n)] = -z\frac{d}{dz}[\frac{z}{z-a}] = \frac{az}{(z-a)^2}$$

4.Z域尺度变换（序列指数加权）

$$X(z) = Z[x(n)], (R_{x1} < |z| < R_{x2})$$

$$Z[a^n x(n)] = X(\frac{z}{a}), (R_{x1} < |\frac{z}{a}| < R_{x2}), (a\text{为非零常数})$$

$$Z[a^n x(n)] = \sum_{n=0}^{\infty} a^n x(n) z^{-n} = \sum_{n=0}^{\infty} x(n)(\frac{z}{a})^{-n}$$

$$= X(\frac{z}{a})$$

$$|z_1| < |z| < |z_2|$$

$$|z_1| < |\frac{z}{a}| < |z_2|$$

例： $Z[\cos(w_0 n)u(n)] = \dfrac{z(z-\cos w_0)}{z^2 - 2z\cos w_0 + 1}, (|z|>1)$

$$Z[\beta^n \cos(w_0 n)u(n)] = \frac{\frac{z}{\beta}(\frac{z}{\beta} - \cos w_0)}{(\frac{z}{\beta})^2 - 2(\frac{z}{\beta})\cos w_0 + 1}$$

$$= \frac{z(z-\beta\cos w_0)}{z^2 - 2\beta z\cos w_0 + \beta^2}, (|\frac{z}{\beta}|>1, \text{即} |z|>|\beta|)$$

5、初值定理

– 若x(n)为因果序列

$$x(0)=\lim_{z\to\infty}X(z)$$

$$\lim_{z\to\infty}X(z)=\lim_{z\to\infty}[\sum_{n=0}^{\infty}x(n)z^{-n}]$$

$$=\lim_{z\to\infty}[x(0)+x(1)z^{-1}+x(2)z^{-2}+\cdots]=x(0)$$

6、终值定理

– 如果x(n)存在终值

$$\lim_{n\to\infty} x(n) = \lim_{z\to 1}[(z-1)X(z)]$$

$$Z[x(n+1)u(n) - x(n)u(n)]$$

$$= zX(z) - zx(0) - X(z)$$

$$= (z-1)X(z) - zx(0)$$

$$(z-1)X(z) = zx(0) + Z[x(n+1)u(n) - x(n)u(n)]$$

$$\lim_{z\to 1}[(z-1)X(z)] = x(0) + \lim_{Z\to 1}\sum_{n=0}^{\infty}[x(n+1) - x(n)]z^{-n} =$$

$$x(0) + [x(1) - x(0)] + [x(2) - x(1)] + \cdots = x(\infty)$$

只有当$n \to \infty, x(n)$收敛，才有确定的终值

$(-1)^k \to \dfrac{z}{z+1}$,求$f(\infty)$

按终值定理：$x(\infty) = \lim\limits_{z \to 1}(z-1)\dfrac{z}{z+1} = 0$

实际上结论不正确，$x(\infty)$不存在

例：$x(n)=a^n - 1, X(z) = \dfrac{z}{z-a} - \dfrac{z}{z-1} = \dfrac{z(a-1)}{(z-a)(z-1)}$,求$x(\infty)$

$$x(\infty) = \lim_{z \to 1}(z-1)\frac{z(a-1)}{(z-a)(z-1)} = -1$$

$a \le 1, X(z)$收敛半径小于1

$a \ge 1, X(z)$收敛半径大于1，$n \to \infty$，$x(n)$发散

7、时域卷积定理

$$Z[x(n)] = X(z) \qquad (R_{x1} < |z| < R_{x2})$$

$$Z[h(n)] = H(z) \qquad (R_{h1} < |z| < R_{h2})$$

$$Z[x(n) * h(n)] = X(z) \cdot H(z),$$

$$\max(R_{x1}, R_{h1}) < |z| < \min(R_{x2}, R_{h2})$$

$$Z[x(n) * h(n)] = \sum_{n=-\infty}^{\infty} [x(n) * h(n)] z^{-n} = \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} x(m) h(n-m) z^{-n}$$

$$= \sum_{m=-\infty}^{\infty} x(m) \sum_{n=-\infty}^{\infty} h(n-m) z^{-n} = \sum_{m=-\infty}^{\infty} x(m) H(z) z^{-m} = X(z) H(z)$$

$$x(n) = a^n u(n), h(n) = b^n u(n)$$

$$H(z) = \frac{z}{z-b}, (|z| > |b|)$$

$$X(z) = \frac{z}{z-a}, (|z| > |a|)$$

$$Y(z) = X(z)H(z) = \frac{z^2}{(z-a)(z-b)} = \frac{1}{a-b}(\frac{az}{z-a} - \frac{bz}{z-b})$$

$$|z| > \max(|a|, |b|)$$

$$y(n) = \frac{1}{a-b}(a^{n+1} - b^{n+1})u(n)$$

$$Z[a^{n-1}u(n-1)] = ?$$

$$Z[a^{n}u(n)] = \frac{z}{z-a}, |z| > |a|$$

$$Z[a^{n-1}u(n-1)] = z^{-1}\frac{z}{z-a} = \frac{1}{z-a}$$

8、序列的乘积特性

$$Z[x(n)] = X(z) \quad (R_{x1} < |z| < R_{x2})$$

$$Z[y(n)] = Y(z) \quad (R_{y1} < |z| < R_{y2})$$

$$w(n) = x(n)y(n)$$

$$W(z) = \frac{1}{2\pi j}\oint_{c1} X(\frac{z}{V})Y(V)V^{-1}dV \qquad W(Z) = \frac{1}{2\pi j}\oint_{c2} X(V)Y(\frac{Z}{V})V^{-1}dV$$

C_1为$X(\frac{z}{V})$与$Y(V)$的ROC重叠区域内逆时针的围线

C_2为$X(V)$与$Y(\frac{z}{V})$的ROC重叠区域内逆时针的围线

复卷积定理 $\quad z = e^{j\phi}, v = e^{j\theta}, dv = je^{j\theta}d\theta$

复卷积定理

$$z = e^{j\phi}, v = e^{j\theta}, dv = je^{j\theta} d\theta$$

$$W(z) = \frac{1}{2\pi j} \oint_c X(e^{j(\phi-\theta)}]Y(e^{j\theta})e^{-j\theta} je^{j\theta} d\theta$$

$$= \frac{1}{2\pi j} \int_{-\pi}^{\pi} X(e^{j(\phi-\theta)}]Y(e^{j\theta})e^{-j\theta} je^{j\theta} d\theta$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j(\phi-\theta)}]Y(e^{j\theta}) d\theta$$

周期卷积

$$W(z)=Z[x(n)y(n)]=\sum_{n=-\infty}^{\infty}x(n)y(n)z^{-n}$$

$$=\sum_{n=-\infty}^{\infty}x(n)[\frac{1}{2\pi j}\oint_{c1}Y(z)z^{n-1}dz]z^{-n}$$

$$=\frac{1}{2\pi j}\sum_{n=-\infty}^{\infty}x(n)[\oint_{c1}Y(V)V^{n}\cdot V^{-1}dV]Z^{-n}$$

$$=\frac{1}{2\pi j}\oint_{c1}\sum_{n=-\infty}^{\infty}x(n)(\frac{z}{V})^{-n}\cdot Y(V)V^{-1}dV$$

$$=\frac{1}{2\pi j}\oint_{c1}X(\frac{z}{V})Y(V)V^{-1}dV$$

Parseval定理

例：求$Z[na^n u(n)], |a|<1$

ZT性质：

$$Z[u(n)] = \frac{z}{z-1}, (|Z|>1)$$

$$Z[nu(n)] = -z\frac{d}{dZ}[\frac{z}{z-1}] = \frac{z}{(z-1)^2}, (|z|>1)$$

$$Z[a^n nu(n)] = \frac{\frac{z}{a}}{(\frac{z}{a}-1)^2} = \frac{az}{(z-a)^2}, (|z|>|a|)$$

Z域卷积性质：

$$X(Z)=Z[nu(n)]=\frac{z}{(z-1)^2},(|z|>1)$$

$$Y(Z)=Z[a^n u(n)]=\frac{z}{z-a},(|z|>|a|)$$

$$Z[na^n u(n)]=\frac{1}{2\pi j}\oint_{c2}X(V)Y(\frac{z}{V})V^{-1}dV$$

$$=\frac{1}{2\pi j}\oint_{c2}\frac{z}{(V-1)^2(z-aV)}dV,\begin{cases}|V|>1\\|\frac{Z}{V}|>|a|\end{cases},\therefore 1<|V|<|\frac{Z}{a}|$$

$$Z[na^n u(n)]=\mathrm{Re}\,s[\frac{z}{(V-1)^2(z-aV)}]_{V=1}=$$

$$\frac{d}{dV}(\frac{z}{z-aV})|_{V=1}=\frac{az}{(z-aV)^2}|_{V=1}=\frac{az}{(z-a)^2},(|z|>|a|)$$

$$X(z) = \ln(1 + az^{-1}), (|z| > |a|)$$

$$x(n) = z^{-1}[X(z)]$$

$$-z\frac{d}{dz}[\ln(1 + az^{-1})] = -z\frac{-az^{-2}}{1 + az^{-1}} = \frac{a}{z + a} = \frac{az}{z + a} \cdot z^{-1} = Z[nx(n)]$$

$$\therefore nx(n) = z^{-1}[\frac{a}{z + a}] = z^{-1}[1 - \frac{z}{z + a}] = \delta(n) - (-a)^n u(n) = -(-a)^n u(n - 1)$$

$$或 = z^{-1}[\frac{a}{z + a}] = z^{-1}[\frac{az}{z + a} \cdot z^{-1}] = a \cdot (-a)^{n-1} u(n - 1) = -(-a)^n u(n - 1)$$

$$\therefore x(n) = -\frac{(-a)^n}{n} u(n - 1)$$

7.6 Z平面与S平面的映射关系

- Z平面坐标与S平面坐标关系

$$z = e^{sT} \to s = \frac{1}{T}\ln z \qquad （f_s = 1/T, \omega_s = 2\pi/T)$$

$$s = \sigma + j\omega,$$

$$z = \mathrm{Re}[z] + \mathrm{Im}[z] = re^{j\theta}$$

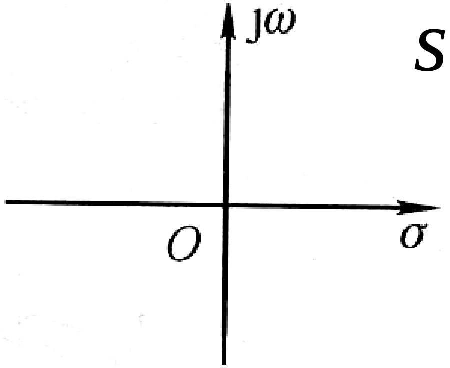
$$= e^{sT} = e^{\sigma T} e^{j\omega T}$$

$$\therefore r = e^{\sigma T}, \theta = \omega T = 2\pi \frac{\omega}{\omega_s}$$

1、虚轴

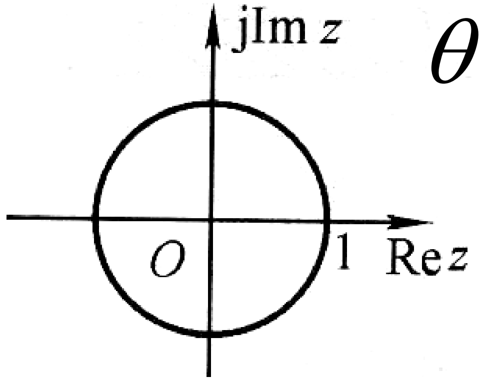
$\sigma = 0$

$s = j\omega$



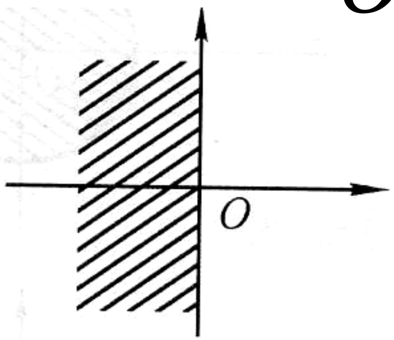
$r = 1, |z| = 1$

$\theta(0 \sim 2\pi)$



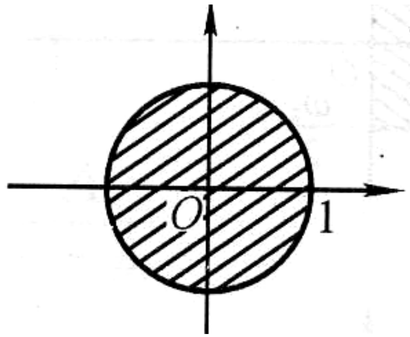
2、左半平面

$\sigma < 0$

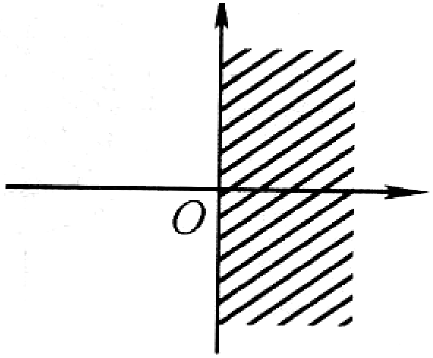


$r < 1$

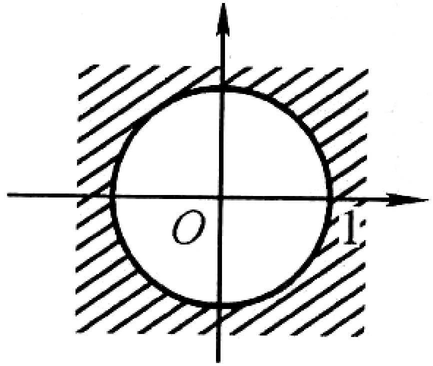
$|z| < 1$



3、右半平面

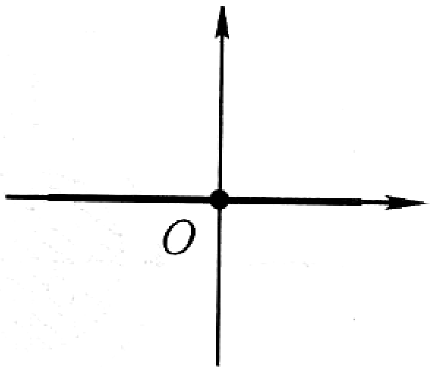


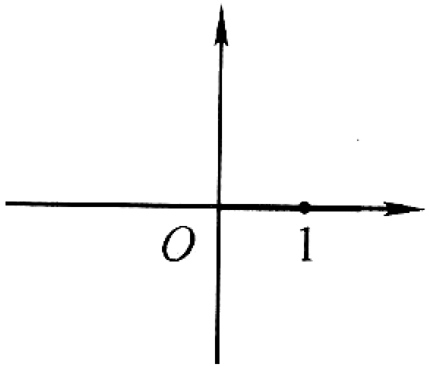
$\sigma > 0$



$|z| > 1$

4、S平面的实轴

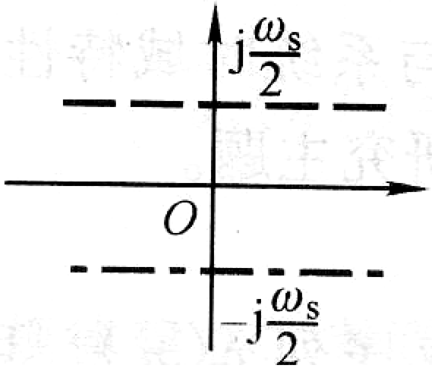




$\omega = 0$

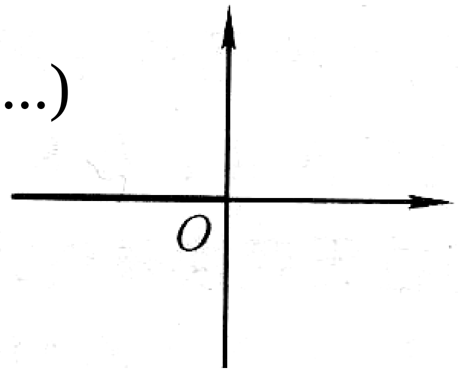
$\theta = \omega T = 0$

5、Z为负实轴

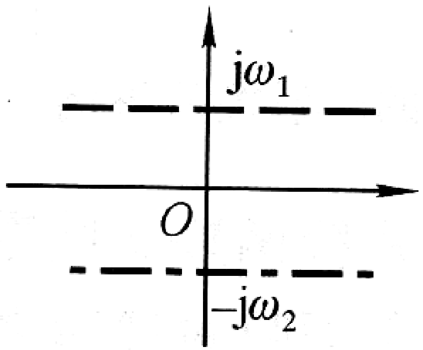


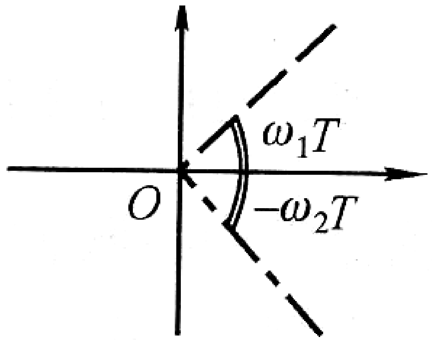
$$\theta = \omega T = k\pi (k = \pm 1, \pm 3 \ldots)$$

$$\omega = \frac{k\pi}{T} = \frac{k}{2}\omega_s$$



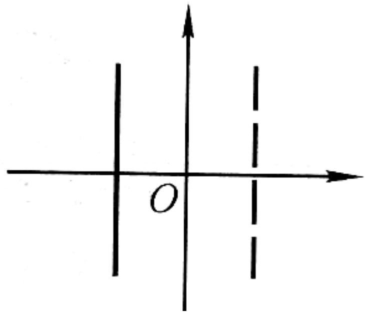
6、平行于实轴的直线

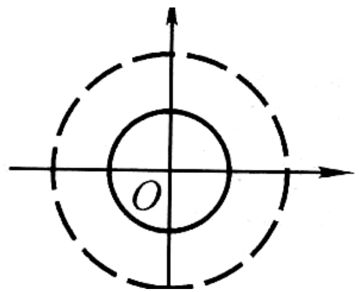




在S平面沿虚轴的移动对应于在Z轴上沿单位圆旋转

7、平行于虚轴的直线





7.7　差分方程的ZT解

1、二阶差分方程的ZT解

$$y(n)+a_1y(n-1)+a_2y(n-2)=b_0x(n)+b_1x(n-1)$$

边界条件$y(-1), y(-2)$

两边同时单边ZT

$$Z[y(n-n_0)u(n)]=z^{-n_0}[Y(z)+\sum_{m=-n_0}^{-1}y(m)z^{-m}]$$

$$Y(z)+a_1[z^{-1}Y(z)+y(-1)]+a_2[z^{-2}Y(z)+z^{-1}y(-1)+y(-2)]$$

$$=b_0X(z)+b_1z^{-1}X(z)$$

$$Y(z)=\frac{b_0+b_1z^{-1}}{1+a_1z^{-1}+a_2z^{-2}}X(z)+\frac{-(a_1+a_2Z^{-1})y(-1)-a_2y(-2)}{1+a_1z^{-1}+a_2z^{-2}}$$

$$=Y_{ZS}(z)+Y_{ZI}(z)$$

如果$y(-1)=y(-2)=0$,即零状态

$$Y(z)=Y_{ZS}(z)=\frac{b_0+b_1z^{-1}}{1+a_1z^{-1}+a_2z^{-2}}X(z)$$

$$=H(z)X(z)$$

2、N阶差分方程

$$\sum_{k=0}^{N} a_k y(n-k) = \sum_{r=0}^{M} b_r x(n-r)$$

$$\sum_{k=0}^{N} a_k z^{-k} [Y(z) + \sum_{l=-k}^{-1} y(l) z^{-l}] = \sum_{r=0}^{M} b_r z^{-r} X(z)$$

$$Y_{zs}(z) = \frac{\sum_{r=0}^{M} b_r z^{-r}}{\sum_{k=0}^{N} a_k z^{-k}} X(z)$$

系统函数 $H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{r=0}^{M} b_r z^{-r}}{\sum_{k=0}^{N} a_k z^{-k}}$

例：$y(n)-5y(n-1)+6y(n-2)=u(n)$

$y(-1)=3, y(-2)=2$，求$y(n), y_{ZI}(n), y_{ZS}(n)$

$$Y(z)-5Z^{-1}Y(z)-5y(-1)+6z^{-2}Y(z)+6z^{-1}y(-1)+6y(-2)=\frac{z}{z-1}$$

$$Y(z)=\frac{1}{1-5z^{-1}+6z^{-2}}\cdot\frac{z}{z-1}+\frac{-[-5y(-1)+6y(-2)+6Z^{-1}y(-1)]}{1-5z^{-1}+6z^{-2}}$$

$$\therefore Y_{ZS}(z)=\frac{z^3}{(z-1)(z^2-5z+6)}=\frac{\frac{1}{2}z}{z-1}+\frac{-4z}{z-2}+\frac{\frac{9}{2}z}{z-3}$$

$$Y_{ZI}(z)=\frac{3-18z^{-1}}{1-5z^{-1}+6z^{-2}}=\frac{3z^2-18z}{(z^2-5z+6)}=\frac{12z}{z-2}+\frac{-9z}{z-3}$$

$$\therefore y_{ZS}(n)=(\frac{1}{2}-4\times 2^n+\frac{9}{2}\times 3^n)u(n), y_{ZI}(n)=(12\times 2^n-9\times 3^n)u(n)$$

$$\therefore y(n)=y_{Zs}(n)+y_{ZI}(n)=(\frac{1}{2}+8\times 2^n-\frac{9}{2}\times 3^n)u(n)$$

7.8　系统函数

1、单位样值响应求系统函数

$$H(z)=\frac{Y(z)}{X(z)}=\frac{\sum_{r=0}^{M}b_r z^{-r}}{\sum_{k=0}^{N}a_k z^{-k}}=G\cdot\frac{\sum_{r=1}^{M}(1-z_r z^{-1})}{\sum_{k=1}^{N}(1-p_k z^{-1})},$$

z_r是$H(z)$的零点，p_k是$H(z)$的极点.

$$Y(z)=H(z)X(z)$$

$$x(n)=\delta(n)\rightarrow X(z)=1$$

$$\therefore h(n)=y(n)=Z^{-1}[H(z)]$$

$$H(z)\ \underset{\text{IZT}}{\overset{\text{ZT}}{\rightleftarrows}}\ h(n)$$

例：$y(n)+3y(n-1)+2y(n-2)=x(n)+x(n-1)$

求$H(z)$, $h(n)$

$$Y(z)+3z^{-1}Y(z)+2z^{-2}Y(z)=X(z)+z^{-1}X(z)$$

$$(1+3z^{-1}+2z^{-2})Y(z)=(1+z^{-1})X(z)$$

$$H(z)=\frac{Y(z)}{X(z)}=\frac{1+z^{-1}}{1+3z^{-1}+2z^{-2}}=\frac{z}{z+2}$$

$$h(n)=(-2)^{n}u(n)$$

2、系统函数的零极点与时间特性的关系

$$H(z)=\frac{\sum_{r=0}^{M} b_r z^{-r}}{\sum_{k=0}^{N} a_k z^{-k}}=G\cdot\frac{\sum_{r=1}^{M}(1-z_r z^{-1})}{\sum_{k=1}^{N}(1-p_k z^{-1})},$$

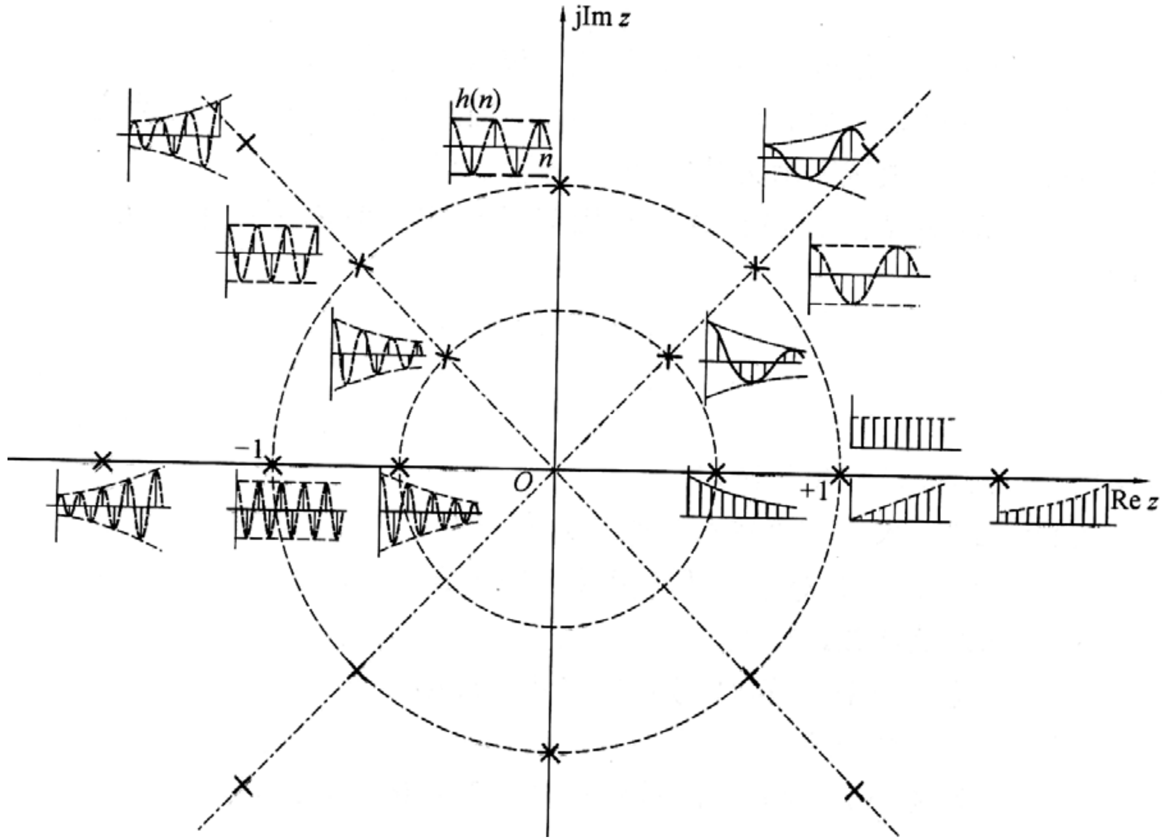
Z_r是$H(z)$的零点，p_k是$H(z)$的极点

$$\frac{H(z)}{z}=\sum_{k=0}^{N}\frac{A_k}{Z-p_k}=\frac{A_0}{Z}+\sum_{k=1}^{N}\frac{A_k}{Z-p_k}$$

$$\therefore H(z)=A_0+\sum_{k=1}^{N}\frac{A_k z}{z-p_k}$$

$$\therefore h(n)=Z^{-1}[H(z)]=A_0\delta(n)+\sum_{k=1}^{N}A_k(p_k)^n u(n)$$

- 极点在单位圆与正实轴的交点上（z=1）,则h(n)为u(n)
- 极点在单位圆上，并以共扼复数形式出现，则h(n)等幅振荡
- 极点在单位圆内，则h(n)指数衰减减幅振荡
- 极点在单位圆外，则h(n)为增幅振荡



3、系统的稳定性

- 系统的稳定性

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty$$

- 极点判断系统的稳定性
  - 极点在单位圆内，系统稳定
  - 极点在单位圆上，系统边界稳定
  - 极点在单位圆外，系统不稳定

例：系统差分方程：$6y(n)-5y(n-1)+y(n-2)=x(n)$

（1）求H(z)，讨论稳定性，收敛性

（2）求$h(n)$,（3）$x(n)=10u(n)$,求$y_{zs}(n)$

$$(1)H(z)=\frac{Y(z)}{X(z)}=\frac{1}{6-5z^{-1}+Z^{-2}}=\frac{z^2}{6z^2-5z+1}=\frac{z^2}{(2z-1)(3z-1)}$$

极点$p_1=\dfrac{1}{2},p_2=\dfrac{1}{3}$,均在单位圆内$\therefore$系统稳定

当$z\to\infty$时,$H(Z)$为有限值,因果系统$\therefore ROC$为$|z|>\dfrac{1}{2}$

$$(2)H(z)=\frac{-1/3}{z-1/3}z+\frac{1/2}{z-1/2}z$$

$$h(n)=[\frac{1}{2}(\frac{1}{2})^n-\frac{1}{3}(\frac{1}{3})^n]u(n)$$

$$(3)Y_{ZS}(z)=H(z)X(z)=\frac{\frac{1}{6}z^2}{(z-\frac{1}{2})(z-\frac{1}{3})}\cdot\frac{10z}{z-1}=\frac{5z}{z-1}+\frac{-5z}{z-\frac{1}{2}}+\frac{\frac{5}{3}z}{z-\frac{1}{3}}$$

$$y_{ZS}(n)=[5-5\times(\frac{1}{2})^n+\frac{5}{3}\times(\frac{1}{3})^n]u(n)$$

7.9　离散系统的频响

1、序列的FT

- 连续时间x(t)的FT
  - X(s)的极点在S平面的左半平面

$$X(j\omega)=X(s)|_{s=j\omega}$$

- 离散时间x(n)的ZT

$$X(z)=\sum_{n=-\infty}^{\infty}x(n)z^{-n}$$

$$x(n)=\frac{1}{2\pi j}\oint_{c}X(z)z^{n-1}dz$$

– S平面虚轴对应Z平面上的单位圆

$$X(e^{j\omega}) = X(z)|_{z=e^{j\omega}} = \sum_{n=-\infty}^{\infty} x(n) e^{-jn\omega}$$

$$x(n) = \frac{1}{2\pi j} \oint_c X(e^{j\omega}) e^{jn\omega} e^{-j\omega} j e^{j\omega} d\omega = \frac{1}{2\pi j} \int_{-\pi}^{\pi} X(e^{j\omega}) e^{jn\omega} d\omega$$

– 离散时间傅氏变换DTFT

$$DTFT[x(n)] = X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n) e^{-jn\omega}$$

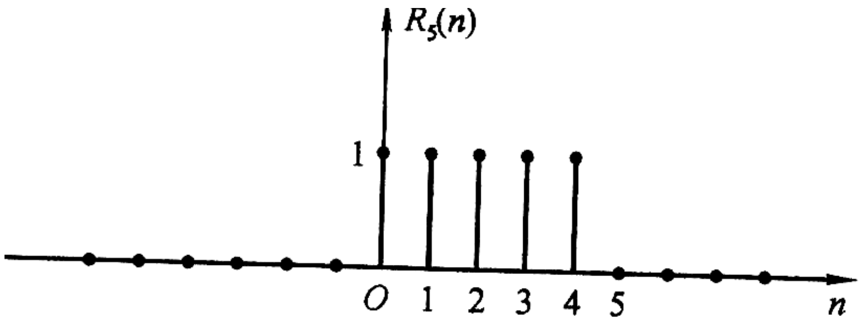
$$IDTFT[X(e^{j\omega})] = x(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) e^{jn\omega} d\omega$$

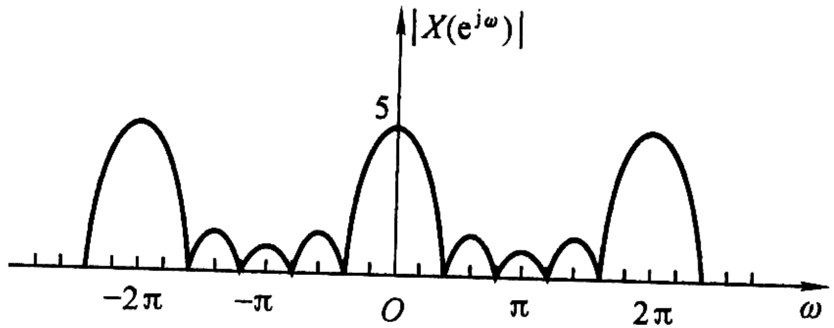
– DTFT的周期性

$X(e^{j\omega})$以2π为周期

例：$x(n)=R_5(n)=u(n)-u(n-5)$,求DTFT

$$X(e^{j\omega})=\sum_{n=0}^{4}e^{-j\omega n}=\frac{1-e^{-5j\omega}}{1-e^{-j\omega}}=e^{-j2\omega}\frac{\sin(5\omega/2)}{\sin(\omega/2)}$$





2、离散系统的频响

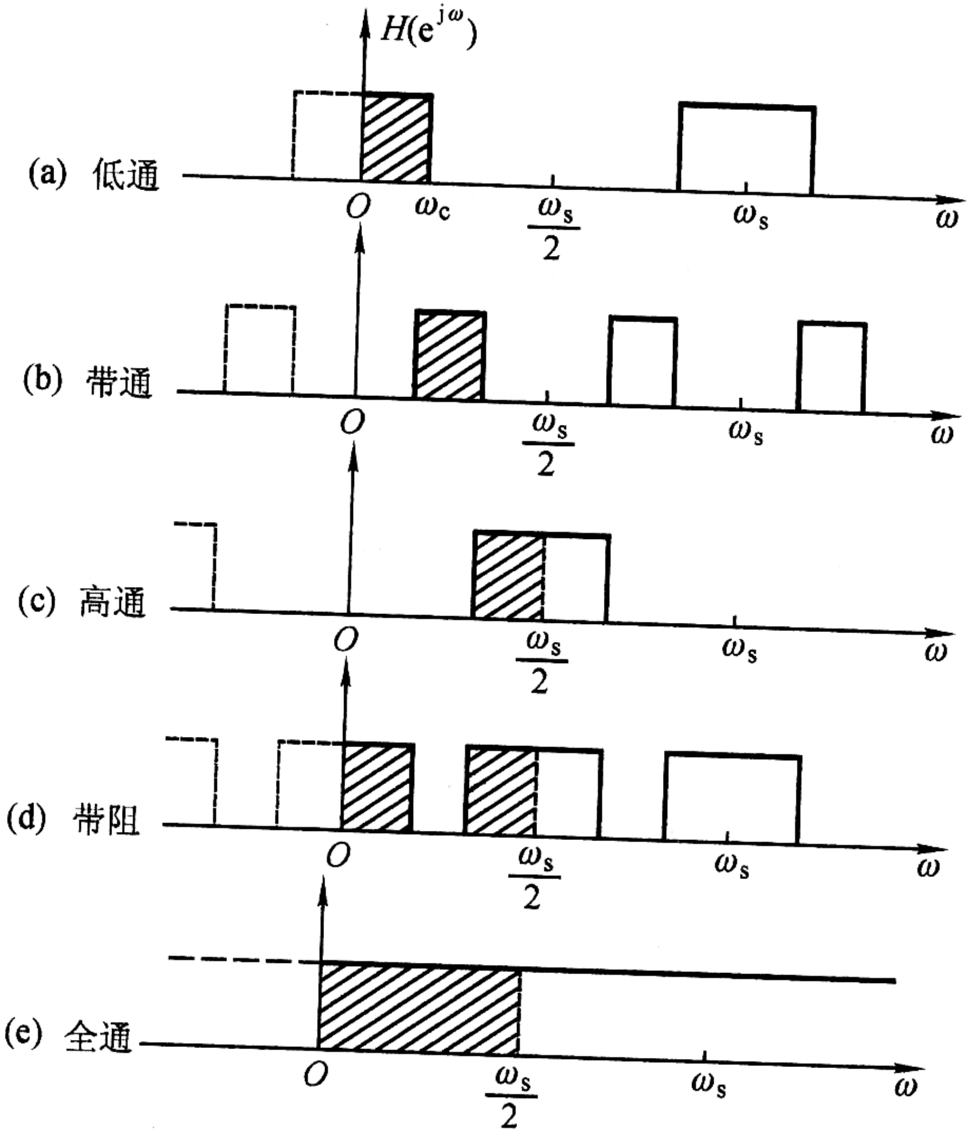
$$H(j\omega) = H(s)|_{s=j\omega}$$

$$H(e^{j\omega}) = H(z)|_{z=e^{j\omega}} = \sum_{n=0}^{\infty} h(n)e^{-jn\omega}$$

$$y(n) = h(n) * x(n) = \sum_{m=0}^{\infty} h(m)e^{j\omega(n-m)}$$

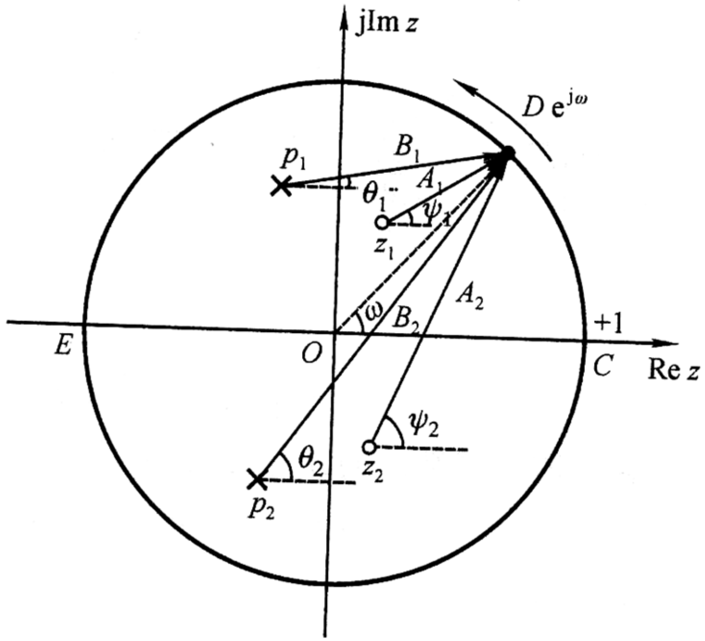
$$= [\sum_{m=0}^{\infty} h(m)e^{-j\omega m}]e^{j\omega n} = H(e^{j\omega})e^{j\omega n}$$

输入信号为虚指或正弦序列时，输入也是一同频的虚指或正弦序列，模及相角被系统函数调制



H(e^{jω})
(a) 低通
O
ωc
ωs/2
ωs
ω
(b) 带通
O
ωs/2
ωs
ω
(c) 高通
O
ωs/2
ωs
ω
(d) 带阻
O
ωs/2
ωs
ω
(e) 全通
O
ωs/2
ωs
ω

3、频率响应的几何确定法



$$H(z)=G\cdot\frac{\prod_{r=1}^{M}(z-z_r)}{\prod_{k=1}^{N}(z-p_k)}$$

$$H(e^{jw})=G\cdot\frac{\prod_{r=1}^{M}(e^{jw}-z_r)}{\prod_{k=1}^{N}(e^{jw}-p_k)}=G\cdot\frac{A_1A_2\cdots A_M}{B_1B_2\cdots B_N}e^{j[(\varphi_1+\varphi_2+\cdots+\varphi_M)-(\theta_1+\theta_2+\cdots+\theta_N)]}$$

例：$H(z)=\frac{z}{z-a},(0<a<1),|z|>a$

试求:(1)$h(n)$,(2)画出方框图（3）画零极点图

（4）求$|H(e^{j\omega})|$和$\varphi(\omega)$,(5)若$x(n)=e^{j\omega n}u(n)$,求$y(n)$

$$(1)h(n)=Z^{-1}[H(z)]=a^n u(n)$$

$$(2)H(z)=\frac{Y(z)}{X(z)}=\frac{z}{z-a}=\frac{1}{1-az^{-1}}$$

$$(1-az^{-1})Y(z)=X(z)$$

$$y(n)=x(n)+ay(n-1)$$

$$(3)H(z)=\frac{z}{z-a},0<a<1$$

$$(4) H(e^{j\omega}) = H(z)|_{z=e^{j\omega}} = \frac{e^{j\omega}}{e^{j\omega}-a} = \frac{1}{1-ae^{-j\omega}} = \frac{1}{1-a\cos\omega + ja\sin\omega}$$

$$|H(e^{j\omega})| = \frac{1}{\sqrt{(1-a\cos\omega)^2+(a\sin\omega)^2}} = \frac{1}{\sqrt{1+a^2-2a\cos\omega}}$$

$$\varphi(\omega) = -tg^{-1}\frac{\sin\omega}{\cos\omega - a}$$

$$(5) X(z) = \frac{z}{z-e^{j\omega}}, |z|>1$$

$$Y(z) = H(z)X(z) = \frac{z^2}{(z-a)(z-e^{j\omega})} = \frac{a}{a-e^{j\omega}}\frac{z}{z-a} - \frac{e^{j\omega}}{a-e^{j\omega}}\frac{z}{z-e^{j\omega}}$$

$$y(n) = \frac{1}{a-e^{j\omega}}[a^{n+1} - e^{j(n+1)\omega}]u(n)$$

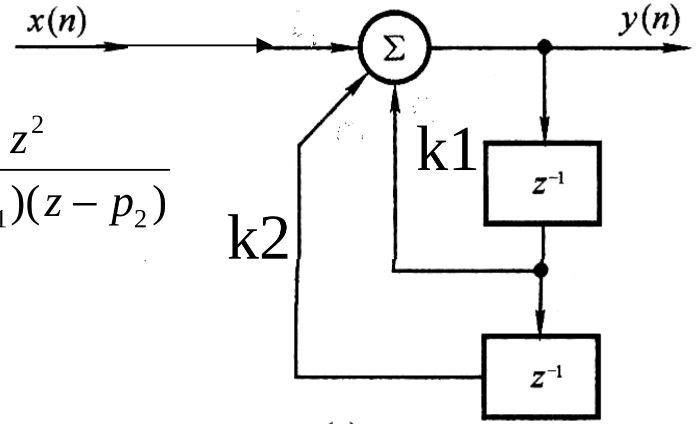
暂态响应　　稳态响应

例：考虑一个二阶（IIR）因果系统：

$y(n)=K_1y(n-1)+K_2y(n-2)+x(n)$，$k_1,k_2$为常数

要求:(1)画方框图（2）求$H(z)$并标出零极点

（3）计算$h(n)$，$n\geq 0(k_1^{\ 2}/4+k_2<0)$



(a)

$$(1)Y(z)=K_1z^{-1}Y(z)+K_2z^{-2}Y(z)+X(z)$$

$$H(z)=\frac{Y(z)}{X(z)}=\frac{1}{1-K_1z^{-1}-K_2z^{-2}}=\frac{z^2}{z^2-K_1z^1-K_2}=\frac{z^2}{(z-p_1)(z-p_2)}$$

当$\frac{K_1^2}{4}+K_2<0$时

$$p_1=re^{j\theta},p_2=re^{-j\theta},\theta=\omega_0T,T=\frac{2\pi}{\omega_s}$$

$$(z-p_1)(z-p_2)=(z-re^{j\omega_0T})(z-re^{-j\omega_0T})=z^2-2r\cos\omega_0Tz+r^2=r^2-K_1z-K_2$$

$$\begin{cases}r^2=-k_2\\2r\cos\omega_0T=k_1\end{cases},\begin{cases}r=\sqrt{-k_2}\\\omega_0=\frac{1}{T}\cos^{-1}\frac{k_1}{2\sqrt{-k_2}}\end{cases}$$

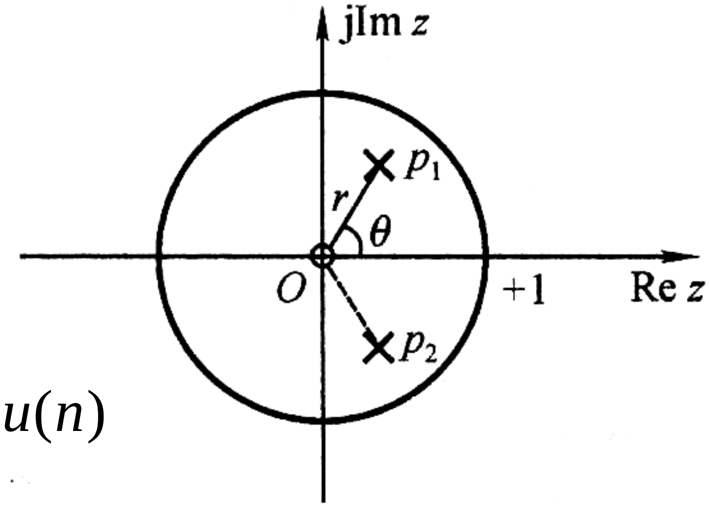
$\omega=\omega_0$时系统共振

$$(3)h(n)=z^{-1}[H(z)]=\frac{1}{2\pi j}\oint_C H(z)z^{n-1}dz$$

$$=\mathrm{Re}\,s[H(z)z^{n-1}]|_{z=p_1}+\mathrm{Re}\,s[H(z)z^{n-1}]_{z-p_2}$$

$$h_1(n)=\mathrm{Re}\,s[H(z)z^{n-1}]|_{z=p_1}=(z-p_1)\frac{z^{n+1}}{(z-p_1)(z-p_2)}|_{z=p_1}=\frac{(re^{j\omega_0 T})^{n+1}}{re^{j\omega_0 T}-re^{-\omega_0 T}}$$

$$=r^n\frac{e^{j(n+1)\omega_0 T}}{2j\sin\omega_0 T}$$



$$h_2(n)=\mathrm{Re}\,s[H(z)z^{n-1}]|_{z=p_2}=r^n\frac{e^{-j(n+1)\omega_0 T}}{-2j\sin\omega_0 T}$$

$$h(n)=h_1(n)+h_2(n)=\frac{r^n}{\sin\omega_0 T}\sin[(n+1)\omega_0 T]u(n)$$

作业

8-1 (6) (7) (8) (9)

8-4

8-5

8-6

8-12

8-19 (1)

8-21 (1) (3) (5)

8-27

8-30